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# IMMC 2018

The 4<sup>th</sup> Annual International Mathematical Modeling Challenge (International)

Summary Sheet

Given the conditions of the hospitals and the situations of themselves, how should people choose the best hospital to go to? We deem that the best hospital should be the one that provides optimal treatment based on one's location, the urgency of the disease, the desire to travel long distances for better medical standards and economic conditions. We come up with two models to address the problem of hospital quality assessment and patient-centered hospital choice.

The first model is to find the evitable mortality of a hospital in order to determine its quality. Employing logistic regression, we build a machine learning model to compute the expected status for each patient. For each disease, we gain the weights of the factors and the bias vector through training and iterations. By comparing a patient's expected status with the actual status, we are able to calculate the number of evitable deaths. Due to the unavailability of patient-specific medical records, we introduce the hospital level-specific average death rate. We regard the average death rate of some disease as a function to the level of a hospital. To verify the model, we computed the rank of the top 20 hospitals in New York.

We then come up with the second model which takes recovery rate, individual choices and patient satisfaction into consideration. Using the similar method, we figure out the recovery rate of each hospital. Individual choices are influenced by one's location, the urgency of the disease, the desire to travel long distances. We render all considerations above into the scoring criteria. In order to combine the factors, we develop 3 different methods to determine the weights of the factors: the multiplication method, the  $\lambda$ -method and Analytic Hierarchy Process (AHP). Then, we test the three methods under scenarios with people of different situations to find the best hospital for them and find that they verify each other. When the scores of two hospitals are 95% close or closer, we take patient satisfaction into consideration by comparing the CMS HCAHPS survey ratings of each hospital.

Finally, we do sensitivity analyses to show the robustness of the models and attached a memo that aids people in choosing the most suitable hospital based on their unique situations.

# A Vital Choice

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# 1 Introduction

## 1.1 Background

Various diseases may pose threat to people's lives, which render hospitals one of the most crucial institutions in the society. In emergency cases, individuals will go to the nearest hospital from their position, when the situation is more severe but less emergent, they tend to seek the hospital that provides best treatment. Also, the economic ability of a person influences his/her affordable cost, so his/her choice will probably be the best treatment one can receive under certain special situation.

Usually, the inpatient mortality is introduced as an indicator to choose the best hospital. But the total number of deaths isn't a proper measurement, because there does exist the situation that the disease or injury is so severe that modern medical technology is unable to cure it. Hence, it's critical to determine the notion of evitable death, which means a death that can be avoided under average medical standard. And what we need to do is to measure the performance of different hospitals by comparing death cases with similar characteristics.

## 1.2 Problem Restatement

Our mission is to find out the factors that influence a hospital's quality and in what ways we can choose the best hospital. Therefore, we divide the problem into 3 parts.

- **Build a model that uses evitable mortality to measure a hospital's quality.**
- **Develop another model measuring the hospital's quality through other factors in addition to evitable mortality.**
- **Test the model's feasibility and sensitivity based on available data.**

## 1.3 Our Work

1. Put forward a model to find the evitable mortality of the hospital.
2. Take more factors into consideration to find out what information a person needs to choose the best hospital.
3. Analyze the model's feasibility and sensitivity based on available data.

## 2 General Assumptions

### 1. High-level hospitals accept patients with more severe diseases.

*According to China Yearbook of Health Statistics, mortality in high-level hospitals are higher than low-level hospitals. That's because people do not go to high-level hospitals when they have slight diseases. The orientation of high-level hospitals are to cure serious diseases.*

### 2. The definition of evitable deaths is that the deaths can be avoided under the average medical standard.

*The bound between inevitable deaths and evitable deaths is quite ambiguous. Humans are still not able to cure certain kinds of severe disease or injury, and they cannot defeat natural mortality, either. And if a hospital doesn't have ability to treat certain patient and transfers he/she to another hospital where he or she can be cured, the death is considered evitable.*

### 3. Particular hospitals are not taken into consideration.

*Particular hospitals include critical access hospitals, long-term care hospitals, children's inpatient facilities, inpatient rehabilitation and psychiatric hospitals, whose mortality rate cannot provide valid information.*

### 4. The best hospital is defined as the optimal treatment a person can receive under his or her conditions and intentions.

*Generally, the best hospital has two meanings. The first one is a hospital offering the optimal treatment for the disease or injury, ignoring all other factors. The second one is the meaning we give above, which is more practical.*

### 5. Under the same conditions, a specialized hospital provides better medical standards than a common hospital. The ability of a normal hospital is average in each speciality.

*Specialized hospitals are usually more professional in certain fields, with more advanced medical devices and sufficient staff. And if a hospital's ability in certain field is quite strong, it will be considered as a specialized hospital.*

### 6. The institution of social insurance and medical insurance of the society is improved.

*In most developed countries, people pay the cost after they are cured. If they do not have the ability, they can take the method of installment. In other situations, they can receive donations from Internet, which can also be regarded as improved social insurance.*

## 3 Model A: Hospital assessments based on evitable mortality

### 3.1 Model Overview

In model A, we intend to rate a hospital by inevitable mortality. In sub-model 1, we introduce machine learning system which requires abundant data to compute evitable deaths. Then we focus

on hospital level-specific average death rates to cope with data insufficiency. By subtracting the evitable mortality from actual mortality, we are able to construct hospital rankings that consider only hospital mortality.

### 3.2 Model Assumptions

1. **All diseases are separately discussed.**

*Each type of disease exhibits specific characteristics which are different from the others, so the respective mappings will also be different. We consider each disease separately to avoid vague classification of illness.*

### 3.3 Variables and Constants

Table 1: **variables and constants of Model A**

Symbol	Definition
$p$	Primary diagnosis of a patient
$DR_p$	The average death rate of primary diagnosis $p$
$AP_p$	All the patients of primary diagnosis $p$ that the hospital received
$AD_p$	Actual deaths of patients of primary diagnosis $p$ that the hospital received
$EM_p$	The rough evitable mortality of primary diagnosis $p$ of the hospital
$M_p$	The mortality of primary diagnosis $p$ of the hospital
$a$	Age of a patient
$g$	Gender of a patient
$u$	Urgency of admission of a patient
$c$	Comorbidity of a patient
$l$	Length of stay of a patient
$\lambda$	The expected status (0 for dead and 1 for alive) of a patient
$\Lambda$	The status (0 for dead and 1 for alive) of a patient
$m$	The final comparative evitable mortality rate of a hospital's department
$W$	The weight matrix
$b$	The bias vector
$DR_p(i)$	The hospital level-specific average death rate of primary diagnosis $p$ for level $i$ hospitals
$DR(i)$	The aggregate death rate of level $i$ hospitals
$DR$	The aggregate death rate of hospitals of the considered area
$n(i)$	The number of level $i$ hospitals
$sDR_p$	The aggregate death rate of primary diagnosis $p$

### 3.4 Pre-modeling discussion

For primary diagnosis  $p$ , the mortality can be calculated by a simple division,

$$M_p = \frac{AD_p}{AP_p} \quad (1)$$

Suppose all hospitals receive patients of the same age, sex, urgency, comorbidity and length of stay distribution as the macroscopic average. The evitable mortality, which reflects how much deaths per unit can be avoided by the average medical level, is,

$$EM_p = M_p - DR_p = \frac{AD_p}{AP_p} - DR_p \quad (2)$$

It can be observed that  $EM_p \in [-DR_p, -DR_p + 1]$ . The lower the value, the finer the hospital performs.

However, by doing this kind of mapping  $M_p \rightarrow EM_p$ , all mortalities are subtracted by a same constant  $DR_p$ , which means that despite the change in values, the relative rankings between the hospitals will not be altered. And if we introduce the process of normalization, the values should be fitted into an interval of  $[0, 1]$ , which means we have to add back the  $DR_p$ , and that renders the calculation of  $EM_p$  futile.

Secondly, each patient has his or her unique properties such as primary diagnosis, age and urgency that together determine the probability of his or her death. And hospitals are also different in many ways. Better hospitals often deal with more severe cases, while other ones mainly treat common cases due to the limitation of technique. The relationship between hospital quality and mortality is shown in the figure below, indicating that in extreme situations the better the hospital quality is, the higher the mortality is.

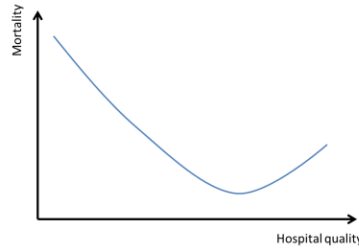


Figure 1: the relationship between mortality and hospital quality

Hence, these calculations are not in accordance with the reality.

### 3.5 Sub-Model 1: Hospital evitable mortality rating based on abundant data

We provide a more effective rating methodology by employing machine learning techniques on patient-specific medical records to compare the quality of hospitals. Firstly, we intend to find out

the relationship between the factors and the status of a patient,i.e.to clarify the function

$$\lambda_p(a, g, u, c, l) \quad (3)$$

which denotes the expected status for a patient of a certain condition in the whole considered area.A patient's expected status with primary diagnosis  $p$  is related to his or her age,gender,admission urgency,comorbidities and length of stay.Hence,the function can be used to calculate the average medical standards in the area.To figure it out,we use logistic regression.

For disease  $p$ ,the expected status should be

$$\begin{aligned} \lambda_p(\mathbf{X}) &= \text{softmax}(\mathbf{X}W + b) \\ \mathbf{X} &= (a, g, u, c, l)^T \end{aligned} \quad (4)$$

where  $\mathbf{X}$  is the input vector,  $W$  is a  $2 \times 5$  matrix of weights and  $b$  a  $2 \times 1$  bias vector.*Softmax* is an exponential normalization function,which transforms all the elements of some vector to fit into an interval of  $(0, 1)$  by the formula

$$\delta(\mathbf{X}_j) = \frac{e^{\mathbf{X}_j}}{\sum_{i=1}^n e^{\mathbf{X}_i}} \quad (5)$$

where  $\mathbf{X}_j$  is the  $j^{\text{th}}$  element of vector  $\mathbf{X}$ .Variables  $W$  and  $b$  are acquired by training.To assess the effectiveness of our training,we introduce the cross entropy cost function

$$C = -\frac{1}{n} \sum_{\mathbf{X}} [\Lambda \times \ln \lambda + (1 - \lambda) \times \ln(1 - \lambda)] \quad (6)$$

where  $n$  is the total number of samples.

After finding  $\lambda_p(\mathbf{X})$ ,we turn to the hospital data again and calculate the expected status of each individual medical record.Those that were expected to live but died,i.e.

$$\begin{aligned} \lambda_p(\mathbf{X}) &= 1 \\ \Lambda_p &= 0 \end{aligned} \quad (7)$$

are counted as evitable deaths,which we denote by  $\alpha_p$ .Let the total death count be  $\beta_p$ ,then the evitable death counts are divided by the total death counts to compute the evitable death rate.

$$m = \frac{\alpha_p}{\beta_p} \quad (8)$$

The nature of this step is to see whether in this aspect the hospital is below the average level of all hospitals considered.The lower the rate,the better and safer the hospital is.The algorithm is shown below,



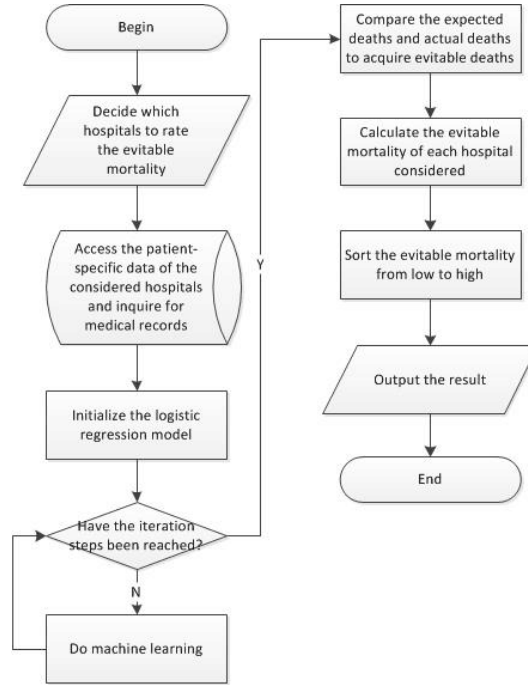


Figure 2: the algorithm of the process to find evitable deaths

### 3.6 Sub-Model 2:Coping with data insufficiency

However, the data requirements in the previous model are considerably difficult to meet. Patient records are confidential data that concerns the privacy of people. In America for example, medical records are considered Personally Identifiable Information (PII) and Protected Health Information (PHI) and are not publically available. The Centers for Medicare and Medicaid Services (CMS) are only willing to reveal Limited Data Sets (LDS) to those who have undergone rigorous and complicated application processes. Therefore, the ideal data required in the sub-model 1 cannot be acquired. Hence, we can only take alternative measures to address the problem.

We introduce the hospital level-specific average death rate of primary diagnosis  $p$ , denote as  $DR_p(i)$ . Then we have,

$$EM_p = \frac{AD_p}{AP_p} - DR_p(i) \quad (9)$$

Where  $i(i = 1, 2, 3, \dots, n)$  measures the level of the hospital. The severer the conditions of patients are, the more they tend to go to higher level hospitals, so in some circumstances, higher level hospitals may have higher death rates. Making the average death rate of some disease a function to the level of a hospital can reflect this kind of situation.

To find  $DR_p(i)$ , we divide the hospital into different levels. We collect the  $DR(i)$  of each level of the hospitals. We can infer that

$$\frac{DR(1)}{DR_p(1)} = \frac{DR(2)}{DR_p(2)} = \dots = \frac{DR(n)}{DR_p(n)} \quad (10)$$

Also, we collect the data of  $DR$  and  $n(i)$ . Given the primary diagnosis  $p$  and its aggregate mortality  $sDR_p$ , we can finally calculate the exact value of  $DR_p(i)$  through

$$sDR_p = \frac{\sum_{i=1}^n n(i)DR_p(i)}{\sum_{i=1}^n DR_p(i)} \quad (11)$$

The level of a hospital is related to its medical ability, including the professional ability of the physicians. It is measured by error rate, the number of treated patients, SCI papers and education background. The education background represents the theoretical level of a physician and can be measured by the degrees one holds and SCI papers one has published. The number of patients the physician has treated and the error rate show his or her practical level. And we can conclude that the level of a hospital also represents the working experience of physicians.

Taking the US as an example, we are able to compute the top 20 rankings of the hospitals in the State of New York. The Hospital Consumer Assessment of Healthcare Providers and Systems (HCAPS) ratings conducted surveys in most hospitals in the United States to give a discrete rough score ranging from 1 to 5, so we classify the hospitals in the US into 5 levels. Applying the model above, we separately calculate the death rates of these 5 groups and compute the evitable death rate. The result of New York are listed as follows.

Table 2: the ranking of top 20 hospitals in the state of New York

Rankings	Hospital name	Evitable mortality(%)
1	NYU HOSPITALS CENTER	-3.96629
2	JOHN T MATHER MEMORIAL HOSPITAL OF PORT JEFFERSON	-3.58432
3	ST FRANCIS HOSPITAL, ROSLYN	-3.38777
4	NEW YORK-PRESBYTERIAN HOSPITAL	-3.34776
5	MAIMONIDES MEDICAL CENTER	-2.95711
6	MONTEFIORE MEDICAL CENTER	-2.80663
7	LENOX HILL HOSPITAL	-2.62017
8	ROCHESTER GENERAL HOSPITAL	-2.56921
9	MOUNT SINAI BETH ISRAEL/PETRIE CAMPUS	-2.30285
10	UNIVERSITY HOSPITAL (STONY BROOK)	-2.30131
11	NYU WINTHROP HOSPITAL	-2.16252
12	MOUNT SINAI HOSPITAL	-2.08989
13	CORTLAND REGIONAL MEDICAL CENTER, INC	-1.95096
14	CANTON-POTSDAM HOSPITAL	-1.93858
15	ST CHARLES HOSPITAL	-1.92914
16	ST JOSEPHS HOSPITAL HEALTH CENTER	-1.88327
17	NEW YORK-PRESBYTERIAN BROOKLYN METHODIST HOSPITAL	-1.78398
18	KINGSBROOK JEWISH MEDICAL CENTER	-1.73865
19	MOUNT SINAI WEST	-1.63536
20	NORTH SHORE UNIVERSITY HOSPITAL	-1.59479

## 4 Model B: The development of comprehensive personalized hospital rankings

### 4.1 Model Overview

To modify the first model, we take more factors into consideration to get the exact information a person needs if he/she wants to find the best hospital in a certain area. The factors cover two main categories. The first one is a view from hospital including the recovery rate. The second one is a view from patients involving the economic situation of an individual and his/her treatment interest that may influence the distance he/she wants to travel. We put forward three methods to determine the weighting of these factors and the conclusions verify each other.

### 4.2 Variables and constants

Table 3: variables and constants of Model B

Symbol	Definition
$r$	The final comparative avoidable recovery rate of a hospital's department
$w$	The comparative medical fees of a hospital's department
$M$	The affordability of a person
$\varepsilon$	The emergency degree of a person
$d$	The desire to travel long distances for fine treatment of a person
$\delta$	The distance between a hospital and a person's place
$f_1$	Scoring function for mortality
$f_2$	Scoring function for recovery
$f_3$	Scoring function for accessibility
$f_4$	Scoring function for affordability
$S$	The overall score of a hospital to a person
$\Omega$	Patient satisfaction index

### 4.3 Determining the influence factors

We do the rankings not only from the hospitals' point of view, but also from the individuals'. There are two ways to assess a hospital: measuring its characteristics, or measure its outcomes.

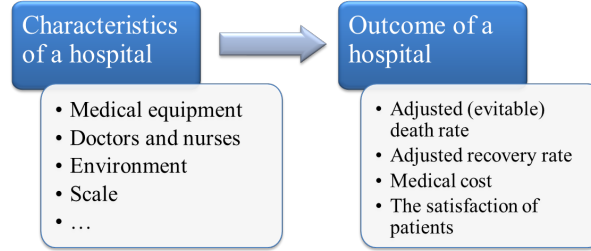


Figure 3: **the relationship between characteristics and outcome of a hospital**

The characteristics of hospitals are often complicated, subjective and trivial. Characteristics are difficult to convert into numbers and it is a credible amount of work to collect data of all the aspects. For example, to gain the academic degrees of all doctors in a hospital is virtually as impractical as gaining the patient-specific medical records.

In contrast with the cause-based evaluation approach, we take the outcome-based approach, which is also theoretically well-grounded and much easier to collect data. All the characteristics of a hospital contribute to its outcome. Therefore, we take the 4 outcome indices, namely evitable death rate, adjusted recovery rate, medical cost and patient satisfaction into consideration. We compute the adjusted recovery index using the preceding method.

Not only should the rankings of hospitals take the conditions of the hospitals into consideration, it should also be aware of the situations of people using it. For those who are financially abundant, the price of treatment of a hospital is of little problem, while those financially challenged may find it hard to deal with. In emergencies one should regard his or her life as the top priority, while when the situation is minor, one may value recovery rate more. Some may be willing to travel long distances to find the best cure while others feel okay to go to hospitals near their place.

Thus we define  $M, \varepsilon, d$  as follows:

Table 4: **the level of urgency, willingness and financial condition**

	$M$	$\varepsilon$	$d$
0.75	wealthy	emergency	willing
0.5	average	slight emergency	slight willing
0.25	poor	no emergency	not willing

Thus, we normalize all data and then render the considerations above into the following scoring criteria,

$$\begin{aligned}
 f_1(m, \varepsilon) &= -\varepsilon m + 1 \\
 f_2(r, \varepsilon) &= (1 - \varepsilon)r \\
 f_3(\delta, d) &= -(1 - d)\delta + 1 \\
 f_4(w, M) &= -(1 - M)w + 1
 \end{aligned} \tag{12}$$

Where we can see that the farther the distance, the less impressive a hospital is to a person, while the desire to travel far can diminish that effect; the higher the cost, the less impressive, while the person's wealth can diminish that effect. The functions  $f_1, f_2, f_3, f_4$  all have a codomain of  $[0, 1]$ .

## 4.4 Combining the factors

Now that we've deduced the scoring function for the four aspects in consideration, namely mortality, recovery, accessibility and affordability, how should we put them together to gain an overall evaluation criterion for the quality of a hospital? This, by its nature, is a subject of profound interest in the field of multi-objective decision making. Here we use several methods to combine the four aspects and compare the results.

### 4.4.1 The Multiplication method

The multiplication method is suitable for pursuing the maximal of the considered objects. We construct the function,

$$S = f_1(m, \varepsilon) \times f_2(r, \varepsilon) \times f_3(\delta, d) \times f_4(w, M) \quad (13)$$

This method has the advantage of not required to weigh the objectives. Objective weighing is mostly empirical and hard to be objectively done. We can observe from above that, once there is a value among  $f_1, f_2, f_3, f_4$  that is close to 0, the total score will be close to 0, which is often the case.

### 4.4.2 The $\lambda$ -method in multi-objective linear programming

We try to find the maximal of each indicator and construct the function,

$$S = \lambda_1 \times f_1(m, \varepsilon) + \lambda_2 \times f_2(r, \varepsilon) + \lambda_3 \times f_3(\delta, d) + \lambda_4 \times f_4(w, M) \quad (14)$$

where

$$\begin{aligned} \lambda_i &= \frac{1}{f_i^0} \\ f_i^0 &= \max f_i(x, y) \end{aligned} \quad (15)$$

And  $x$  and  $y$  should be in the domain of definition which is  $[0, 1]$ . This method is suitable for pursuing maximum of the considered objectives.

### 4.4.3 Analytic hierarchy process

The analytic hierarchy process (AHP) was invented by T. L. Saaty in the 1970s. The method collects the essence of psychology and social sciences to aid decision makers in turning subjective experience into quantitative measures. The problems that the AHP handles have to comprise of three parts, i.e. the objective level, the criteria level and the measures level. For example, the hospital choosing problem can be represented in the following graphs.

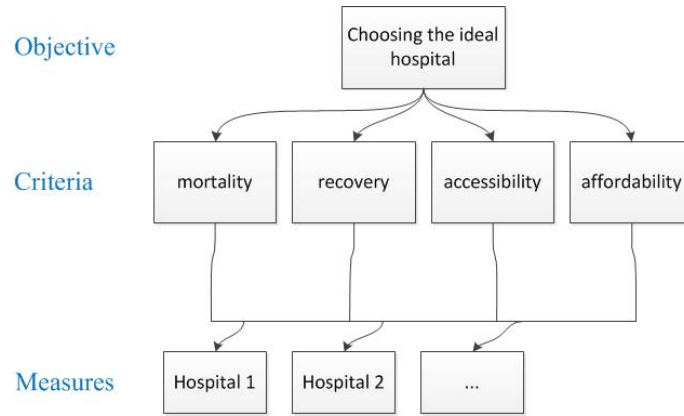


Figure 4: the levels of hospital chosen problem

Now we briefly elucidate how the theory works. Suppose there are  $n$  items  $A_1, \dots, A_n$ , and their weights are respectively  $w_1, \dots, w_n$ . If we intend to compare the weight of each pair of them, we can get an  $n \times n$  matrix

$$A = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \dots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \dots & \frac{w_2}{w_n} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \dots & \frac{w_3}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \dots & \frac{w_n}{w_n} \end{pmatrix} \quad (16)$$

It is the property of  $A$  that

$$A\mathbf{W} = n\mathbf{W} \quad (17)$$

where

$$\mathbf{W} = (w_1, w_2, \dots, w_n)^T \quad (18)$$

is the weight vector and the eigenvector of matrix  $A$  and that  $n$  is the sole real nontrivial maximum eigenvalue of matrix  $A$ . This is because for all elements  $a_{ij}$  ( $i, j = 1, 2, \dots, n$ ) of  $A$ , we have

$$\begin{cases} a_{ii} = 1 \\ a_{ij} = \frac{1}{a_{ji}} \\ a_{kj} = \frac{a_{ij}}{a_{ik}} \end{cases} \quad (19)$$

However, in reality, it is hard to ensure that the equations above stand. So we employ a scoring mechanism for the matrix. The decision of  $a_{ij}$  can be made through the following chart.

Table 5: **an evaluation standard of AHP**

$a_{ij}$	Rules
1	$i$ and $j$ are of the same importance
3	$i$ is slightly more important than $j$
5	$i$ is more important than $j$
7	$i$ is much more important than $j$
9	$i$ is definitely more important than $j$
2, 4, 6, 8	Cases in the middle
Reciprocals of above	Comparing $j$ with $i$

This is in light of the psychological fact that the limit of information level determination of human beings is within  $7 \pm 2$  levels.

Based on the discussion above, we here design the  $A - C$  and  $C_i - P (i = 1, 2, 3, 4)$  determination matrix. Because all criteria's weights to the measures are the same (the measures are all of the same kind, i.e. hospitals), all elements of the  $C_i - P$  matrix are 1.

Table 6: **the weight of the system**

A	$C_1$	$C_2$	$C_3$	$C_4$
$C_1$	1	1	9	7
$C_2$	1	1	7	5
$C_3$	$\frac{1}{9}$	$\frac{1}{7}$	1	$\frac{1}{5}$
$C_4$	$\frac{1}{7}$	$\frac{1}{5}$	5	1
A	$P_1$	$P_2$	$\dots$	$P_9$
$P_1$	1	1	$\dots$	1
$P_2$	1	1	$\dots$	1
$\vdots$	1	1	$\dots$	1
$P_9$	1	1	$\dots$	1

Thus it can be computed that

$$W = (4.35022, 3.70103, 0.374672, 1.0000)^T \quad (20)$$

So that

$$S_{AHP} = 4.35022 \times f_1(m, \varepsilon) + 3.70103 \times f_2(r, \varepsilon) + 0.374672 \times f_3(\delta, d) + 1.0000 \times f_4(w, M) \quad (21)$$

The AHP is a highly flexible and interactive method, so the relative importance is not fixed and can even be modified by the user of the ratings if they wish.

## 4.5 Considering patient satisfaction

Furthermore, we take patient satisfaction into consideration. The value of the index  $\Omega$  can be found in the CMS HCAHPS ratings consists of an overall rating summarized by patients who took

the surveys of CMS. The scores are then normalized into  $[0, 1]$ . The HCAHPS Survey contains 21 patient perspectives on care and patient rating items that encompass nine key topics, which is used by assessing the percentage of respondents and can be found in local health profiles.

Here we employ the method of mathematical programming to add patient satisfaction into consideration. Satisfaction is less important than the 4 aspects above, so if the  $S$  of 2 hospitals are close (for example 95% close or closer as it is often used in statistics), we take satisfaction into account. The one with the higher satisfaction is rated higher. If the  $S$ s are far, then we do not need to consider satisfaction any more.

## 4.6 Model application

Now it is entirely feasible to rank hospitals for a person wherever he or she is and whatever his or her situation may be based on a medical database and some calculations through our model. Here we take Manhattan for example and rated the 9 hospitals in the area in several scenarios.

First, we rate them simply by evitable mortality using model A:

Table 7: the result of model A

Color	Rank	Hospital Name	Evitable mortality (%)
Blue	1	NYU HOSPITALS CENTER	-3.96629
Cyan	2	NEW YORK-PRESBYTERIAN HOSPITAL	-3.34776
Green	3	LENOX HILL HOSPITAL	-2.62017
Black	4	MOUNT SINAI BETH ISRAEL/PETRIE CAMPUS	-2.30285
Purple	5	MOUNT SINAI HOSPITAL	-2.08989
Red	6	MOUNT SINAI WEST	-1.63536
White	7	METROPOLITAN HOSPITAL CENTER	-1.2279
Yellow	8	HARLEM HOSPITAL CENTER	-0.19184
Orange	9	BELLEVUE HOSPITAL CENTER	0.966695

In the following demonstration of results, we use our defined color of each hospital to mark which hospitals the patients are recommended to go to. In fact, for every point on the map, we can calculate the hospital's rankings based on the 3 methods and even sort all the non-inferior solutions (a non-inferior solution is one in which an improvement in one objective requires a degradation of another).

### 4.6.1 Cases of no emergency, average wealth and no desire to travel long distances

In this situation, the scores of  $M, \varepsilon, d$  are respectively 0.5, 0.25, 0.25. We apply the methods  $a, b$  and  $c$  and gain the following results.





Figure 5: the ideal hospital for cases of no emergency, average wealth and no desire to travel long distances

#### 4.6.2 Cases of emergency, average wealth and no desire to travel long distances

In this situation, the scores of  $M, \varepsilon, d$  are respectively 0.5, 0.75, 0.25.

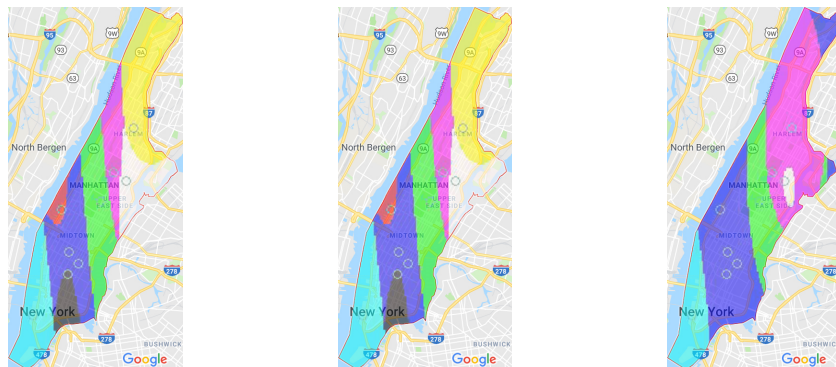


Figure 6: the ideal hospital for cases of emergency, average wealth and no desire to travel long distances

#### 4.6.3 Slight emergency, wealthy and with desire to travel long distances

In this situation, the scores of  $M, \varepsilon, d$  are respectively 0.75, 0.5, 0.75.

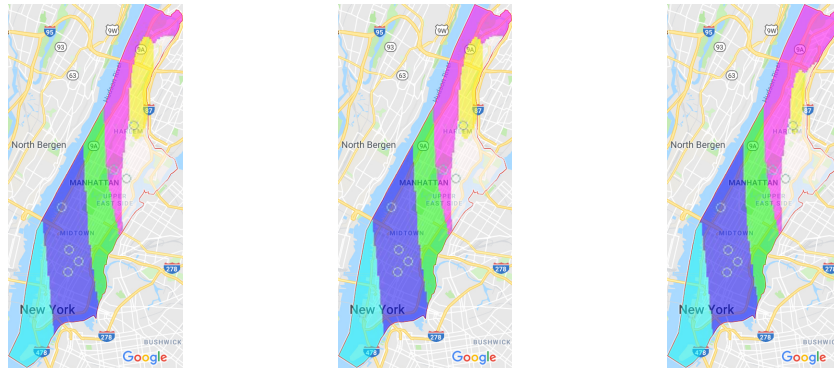


Figure 7: **the ideal hospital for cases of slight emergency, wealthy and with desire to travel long distances**

We can see from above that the result of the three methods we employ are similar and confirm with each other. This proves that the weightings we choose are mostly reasonable and our model is robust. Apart from these typical scenarios, we can also rate the hospitals under other circumstances. Not only can we create the user-based hospital map in Manhattan, but also we can map New York, America and even the world given relevant data. It is also feasible now that we build a graphic user interface to enable greater visualization.

## 5 Sensitivity Analysis

We use AHP as the third method of our model which includes subjective factors. Hence we put forward our sensitivity analysis of the weights and the results.

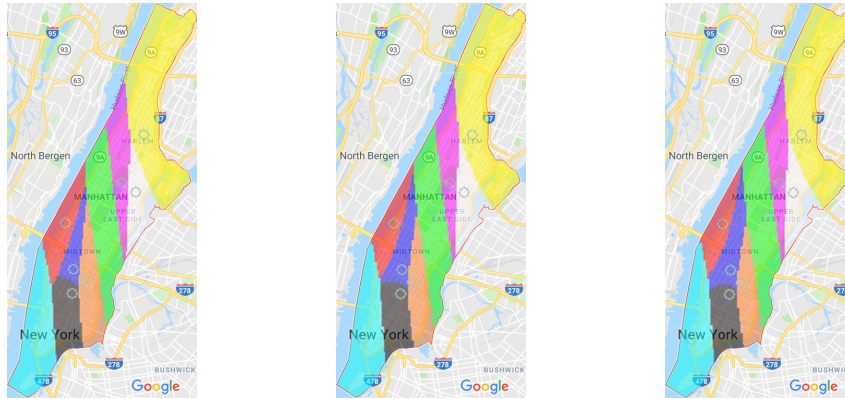


Figure 8: the ideal hospital for cases of no emergency, not wealthy and with desire to travel long distances

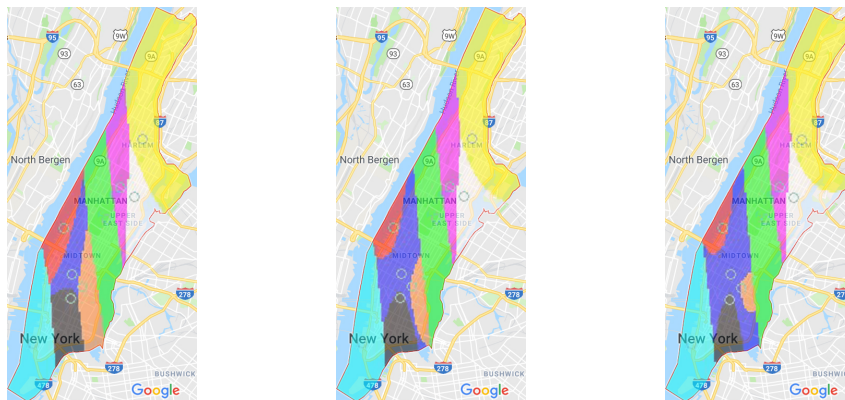


Figure 9: the ideal hospital for cases of slight emergency, not wealthy and with desire to travel long distances

Though the weights have changed, the results still confirm with each other, which testifies the stability of our model.

## 6 Strengths and Weaknesses

### 6.1 Strengths

1. **Practicality**

Normal hospital evaluations are based on the characteristics of hospitals. We not only consider the hospital features, but also the view from patients and individuals, showing high practicality.

2. **Universality**

In the test of our model, we find it with high universality that can be used in a wide range of hospitals to evaluate their quality.

### 6.2 Weaknesses

1. **Complexity**

Detailed and precise, our mathematical model requires great complexity, with calculations about the weights of each factor.

## 7 A Two-page Memo

**TO:** Any people puzzling at how to choose the best hospital

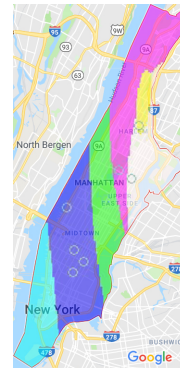
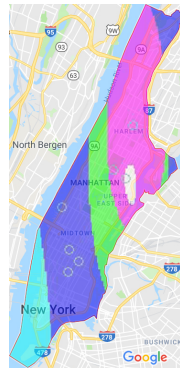
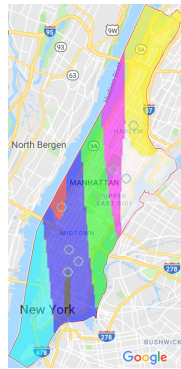
Health is always a foremost concern of people and the choice of hospitals are of great importance. Nevertheless, there are various criteria and respects of hospitals and it is often hard to decide. Also, there is a plethora of data on the net but some information is limited or not collected. In light of the status quo, we come up with a model to determine the rankings of hospitals. The model we developed not only takes the data of the hospitals into consideration, it is also aware of your conditions so that it can be more people-centered.

In our construction of the model, we include 4 factors, namely mortality, recovery, accessibility and affordability. Due to the variance in level and size of hospitals, the mortality we compute is the *evitable* mortality, which reflects more the medical standards of hospitals. For example, the *evitable* mortality rankings of Manhattan, New York can be listed as follows.

Color	Hospital Name
Blue	NYU HOSPITALS CENTER
Cyan	NEW YORK-PRESBYTERIAN HOSPITAL
Green	LENOX HILL HOSPITAL
Black	MOUNT SINAI BETH ISRAEL/PETRIE CAMPUS
Purple	MOUNT SINAI HOSPITAL
Red	MOUNT SINAI WEST
White	METROPOLITAN HOSPITAL CENTER
Yellow	HARLEM HOSPITAL CENTER
Orange	BELLEVUE HOSPITAL CENTER

Some personal conditions of yourself need to be inputted to generate the final rankings, namely your location, urgency of the disease, the desire to travel long distances for better medical standards and economic conditions. Each condition has three discrete values, which are low, medium and high.

Once the simple input is done, our model will generate a hospital ranking. For instance, we analyze the 9 hospitals of Manhattan, New York. If you have no emergency, average wealth and no desire to travel long distances, the recommended hospital depending on your location is shown in the leftmost map. If you are in emergency, and have average wealth and no desire to travel long distances, the recommendations are shown in the map in the middle. If you have slight emergency, are wealthy and have desire to travel long distances, the recommendations are shown in the map on the right. The colors on the map refer to hospitals in the table above.



If none of the scenarios matches your conditions, you can adjust the inputs yourself to see the map most suitable for you.

Our data sources include the Centers for Disease Control and Prevention, the Center for Medicare and Medicaid Services, [data.medicare.gov](http://data.medicare.gov), QualityNet, [HealthData.gov](http://HealthData.gov), Healthgrades and Research Data Assistance Center, so you may rest assured that the decisions we make are accurate and well-grounded.

Then, considering hospital quality, you can ask the hospital some specific, probing questions like

- **Do you have guidelines for everyone getting the surgery I am about to receive?**

This indicates the hospital's responsiveness of patients and their theoretical and practical knowledge of certain kind of disease.

- **How many nurses are in the department, and how many patients do they handle?**

Human resource is an important part to determine the service of a hospital.

- **How often have surgeries like mine been performed?**

General hospitals handle a wide range of routine conditions. Specialty hospitals have a lot of experience with certain conditions or certain groups. Under the same condition, we recommend you to go to specialty hospitals. Hospitals that do many of the same types of procedures tend to have better chances of success with them. So knowing how often the procedure is done there can help you make the choice.

We hope that our ratings can bring conveniences and benefits to you. We would be much encouraged and delighted if you are inspired by our work in ranking the hospitals. Thank you for your reading of this memo.

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- [10] Yale New Haven Health Services Corporation Center for Outcomes Research and Evaluation, *Medicare Hospital Quality Chartbook 2012*

## Appendix

### Data sources

- Centers for Disease Control and Prevention (<https://www.cdc.gov/>)
- Center for Medicare and Medicaid Services (<https://www.cms.gov/Research-Statistics-Data-and-Systems/Research-Statistics-Data-and-Systems.html>)
- data.medicare.gov (<https://data.medicare.gov/>)
- QualityNet (<http://www.qualitynet.org/>)
- HealthData.gov (<https://www.healthdata.gov/>)
- Healthgrades (<https://www.healthgrades.com/>)
- Research Data Assistance Center (<https://www.resdac.org/>)
- China National Health and Family Planning Commission (<http://www.nhfpc.gov.cn/>)