2018 IM^{2}C
Summary Sheet

Team Control Number
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Roughly 33 million patients check into American hospitals every year, at a rate of about 1 patient per second. Nearly everyone in their lives will try to seek healthcare services, and they will try their best to find the "perfect" hospital for themselves. How to determine whether a hospital is good or not then becomes a question. They also have their own choices and opinions about healthcare services. So, how can we measure the quality of a hospital?

Certainly, mortality is a crucial variable — no-one wants to go to hospitals where doctors can't cure patients. Other factors are also very important, as evaluations need to be well-rounded so patients can truly know the overall quality of a hospital before they go there. In this paper, **we considered four factors: mortality, patient ratings, patient safety, and cost to evaluate the quality of American hospitals, then developed two models (AHP and PWSCM) to rank national and local hospitals**. Lastly, **we developed a software service (Hospital Evaluation Ranking System — HERS) that can use patient's opinions and their location to find the best hospital they should choose in different circumstances.**

We built a mortality model (Comparative Mortality Model, CMM) based on disease-caused mortality and operation-caused mortality, which takes into account death distributions in each state, and comparing each hospital's mortality with the state average.**This model ranked the CEDARS-SINAI MEDICAL CENTER as the best hospital.**

Using similar methods, we analyzed the patient ratings, patient safety, and cost of every hospital in the nation, comparing each measure to the state average. From these factors and mortality, we built an AHP model to rank the best hospital in the nation, but found the algorithm inefficient. Based on the AHP model, we developed our own model, called Patient-Weighted State Comparison Model, which can efficiently and accurately rank all the hospitals in the nation. **Both the AHP model and our model ranked MAYO CLINIC HOSPITAL as the best hospital in the United States.** We also compared our own national rankings with the U.S. News 2017-18 Best Hospitals Honor Roll and found it mostly similar but with a few differences which are caused by the different criteria and ranking methods.

Using the PWSCM method, we built a software (HERS) that can acquire the patient's opinion of the importance of each factor on a scale on 1-10, then using the patient's zip code, show the highest ranked hospitals near the patient in different situations (Emergency, Non-emergency…). **Using Los Angeles as a case study, we found that a patient living near downtown should go to either Ronald Reagan UCLA Medical Center or Centinela Hospital Medical Center, depending on the situation.**

Although there are some limitations in our model, for it does not include hospital size, staffing, and specific diseases and specialties within hospitals, it is still very adaptable and can be used in different countries once the accurate data are provided. Our model can be used extensively in real-world scenarios, and can give crucial information to patients. It is also very user-friendly, giving the patient much control over their personalization, but encapsulating the important ranking algorithms. We strongly recommend our model because of its accuracy and adaptability, and ease of use.

HERS

TO: Dear HERS Users

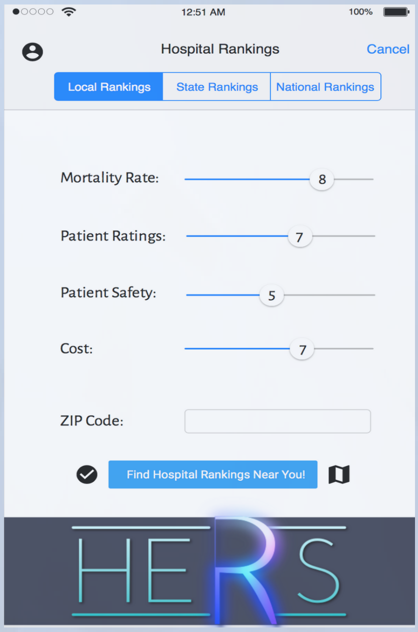
FROM: HERS original developers

SUBJECT: User instructions

WELCOME TO THE INSTRUCTION PAGE OF HERS!

Our Hospital Evaluation Ranking System (HERS) uses four major indicators to help you develop customized hospital rankings.Using our hospital rankings and our software, you can find the most ideal hospital for your condition.

To help us develop your personalized hospital rankings, you need to **determine the importance for each of the indicators on a range of 1-10.** The more important you think each factor is crucial to your hospital experience, the higher you should rate it.



The indicators are:

- √ Mortality Rate
- √ Patients' Rating
- √ Patient Safety
- √ Cost

Our software can compare hospital in three different ranges:

- √ Local Rankings
- √ State Ranking
- √ National Rankings

You should select one of the ranges, so that we can tailor our rankings to you preferences.

If you selected the Local Rankings or State Rankings, you should also provide your ZIP code, so that we can help you find best hospitals NEAR YOU!

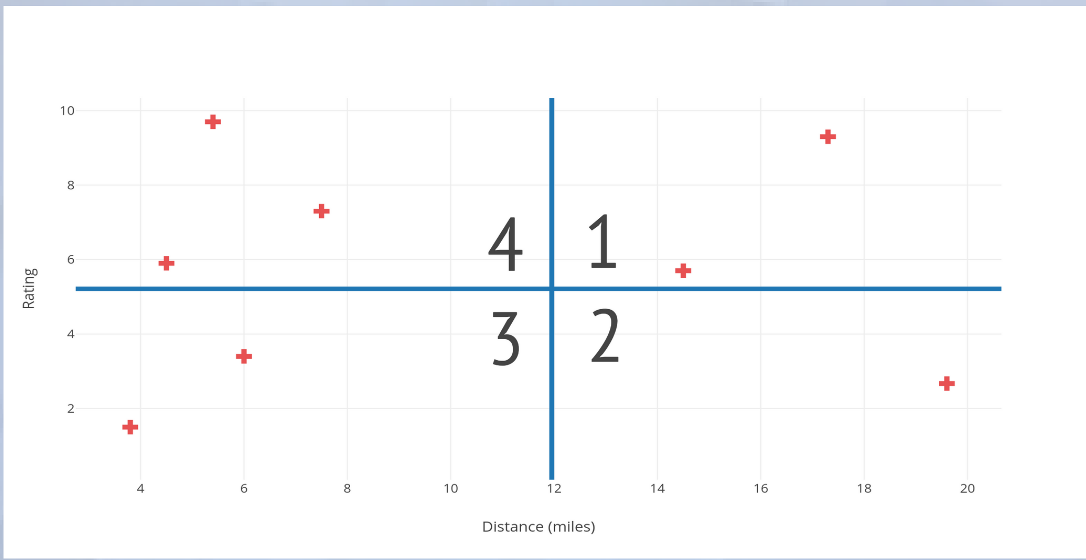
MAY GOD BLESS YOU
FOR A HEALTHY LIFE

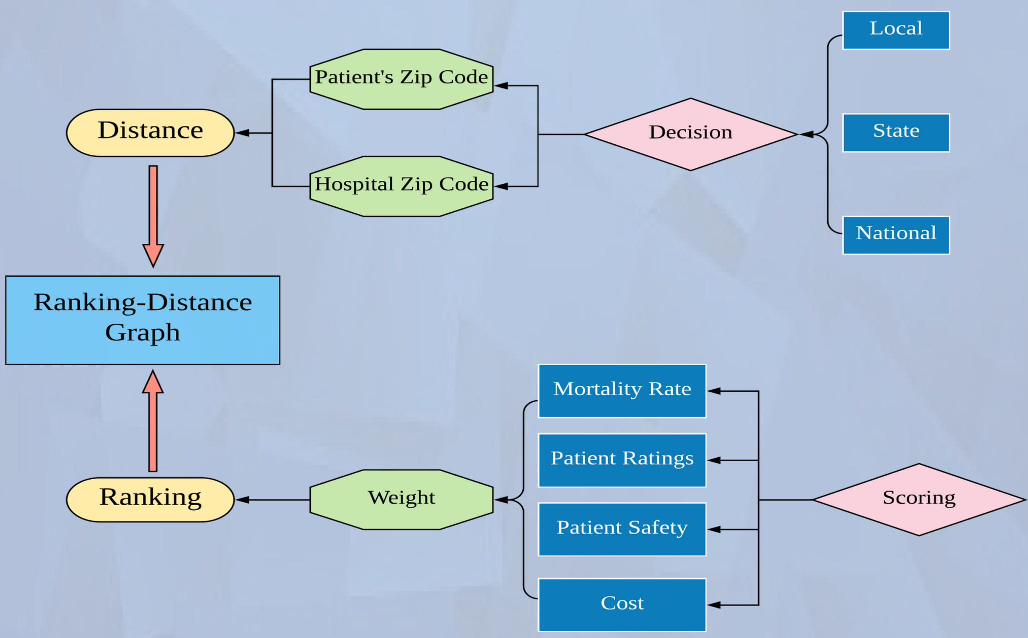
HOW TO USE HERS R-D GRAPH TO CHOOSE AN IDEAL HOSPITAL?

After inputting your preferences and location, **HERS will present youa Ranking-Distance Graph** (R-D Graph). Using the R-D Graph, you caneasily find the most ideal hospitals for you.

☞ If you are **in an emergency**, hospitals far away (Regions "1" or "2") are not good choices. Due to the need for treatment, we strongly recommend you to select **hospitals in regions "3" or "4"**. The best hospitals are location in region "4", so you should prioritize hospitals in that region.

☞ If you have a **chronic disease**, a better treatment is more crucial. Therefore **hospitals of "1" or "4"** would be a good choice.





MAY GOD BLESS YOU FOR A HEALTHY LIFE

1 Introduction

For the sake of maintaining health, people try their best to seek the "perfect" health care system for themselves, but how to determine whether a hospital is good or not becomes a serious question. Sometimes, in emergencies, individuals will most likely seek the closest hospital to them, but many times people will want to find the "best" hospital that is around them. Without extensive data or research, this is a very hard thing to achieve.

A hospital may have a satisfying service, but their mortality rate or cost might be very high. Other hospitals will have a low mortality rate, but they may have staff shortages and unclean environments, among other things.

To help people solve this difficult but important decision-making problem, we developed a system that can rank hospitals near the patient's address, analyzing factors such as mortality rate, patient safety, cost, and other patients' ratings. The individual may self-determine the relative importance of each factor, or use our pre-determined weights. Then the individual can choose the best hospital for their situation, whether choosing a closer hospital or a farther hospital with higher rankings.

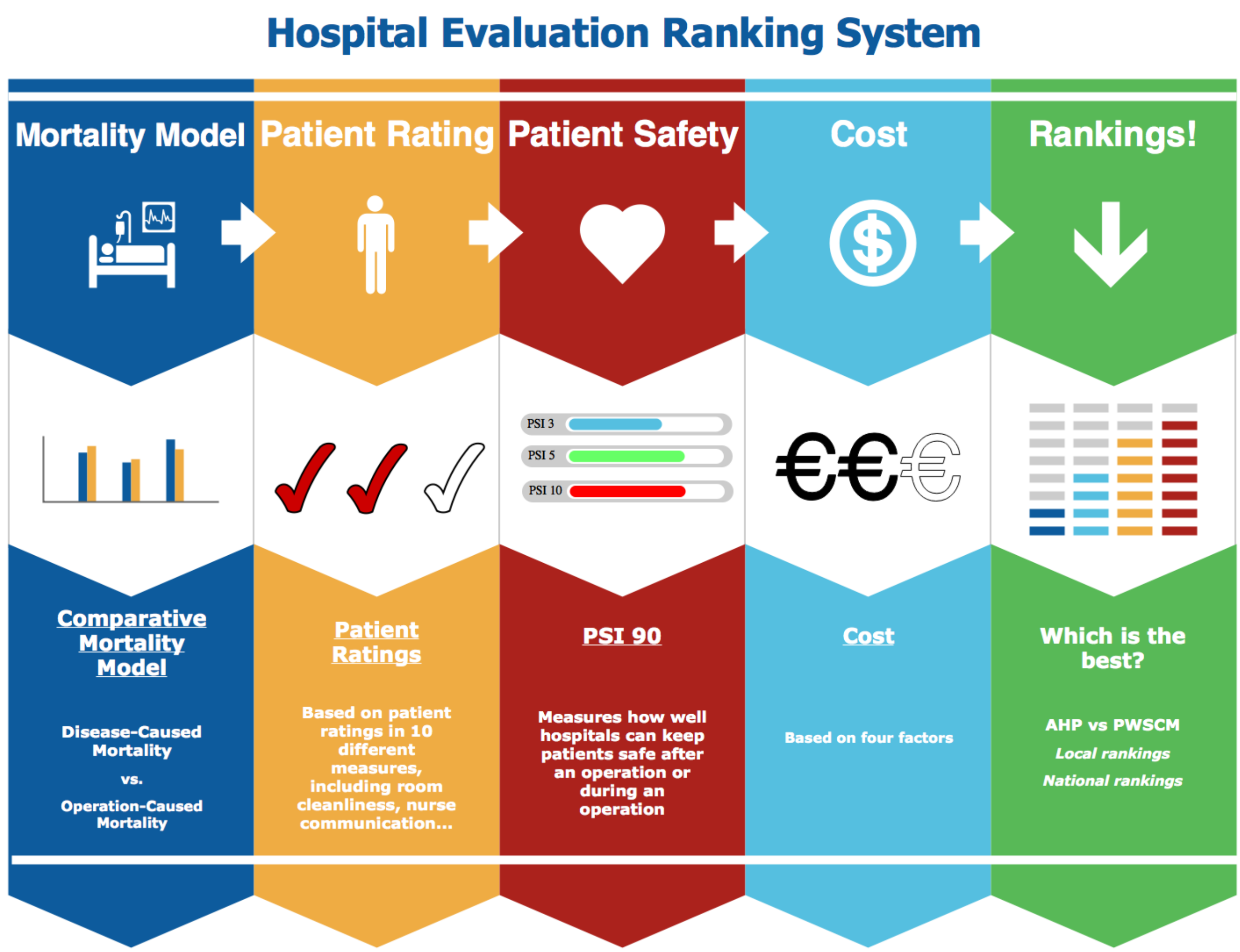


Figure 1: Hospital Evaluation Ranking System

2 Problem Restatement

The question asks us to develop a model for a patient to choose the best hospital using mortality rate among other factors. Therefore, we are required to:
✓ Develop a mathematical model to measure the quality of hospitals using mortality rate.
✓ Develop a mathematical model that uses other factors in addition to mortality to measure the quality of a hospital, then rank the best hospitals.
✓ Write a user-friendly memo that allows patients who lack mathematical expertise and computing ability to be able to choose a hospital that is suitable for them.

3 General Assumptions and Justifications

⇝ The state average for a measure can represent hospitals without data in that measure
Justification: Many smaller hospitals within the United States have a smaller dataset to work with, as they lack the score for a measure. For these cases, an approximation of the hospital quality is the state/national average for that measure. This can allow all the hospitals to be analyzed and ranked.

⇝ Patient surveys are accurate and unbiased
Justification: A large part of our ranking system is the user experience at that hospital, collected from surveys[1] describing the hospital service from 10 factors. We assumed that the results from these surveys are accurate, and correctly reflect the conditions at that hospital. If the data were biased, then it would be almost impossible to know the actual circumstances of hospitals without direct observation.

⇝ The data collected from each hospital is complete and truthful
Justification: Since all the data we collected were from government-affiliated organizations and departments, we assumed that hospitals are truthfully reporting their data, including its mortality rate. As with patient experience, we assumed that no hospitals secluded important information that can drastically change their ranking, and gave complete and honest information to health services.

⇝ All the healthcare data are authoritative, and can effectively represent the overall condition at each hospital
Justification: We obtained data from Medicare.gov and the NIS database [2], among other sources, all of which were deemed authoritative. We can assume that by using these information, a hospital's overall quality can be derived.

⇝ Deaths by certain diseases are evenly distributed within each state
Justification: Deaths by different conditions (e.g. Heart Attack, stroke) vary from state to state, with some states' residents having a probability of a heart attack much greater than other states'. We used this data to adjust the mortality rate of different hospitals, located in different parts of a state. If the death rates by disease in different parts of the same state varies, it would be very hard to calculate.

⇝ There are no significant medical breakthroughs after the data was collected (January 2018)
Justification: Although the data was collected relatively recently (this year), a huge medical breakthrough in a procedure or condition would disrupt the mortality rate and greatly affect our rankings.

⇝ The overall quality of a hospital can be qualitatively modeled by several quantitative measures.
Justification: Since we have to compare the qualities of different hospitals, we need to quantify the "quality" of each hospital. Therefore, we shall assume that this quantity can be quantitatively derived from different measures, or else our model would be rendered ineffective.

⇝ Special hospitals are able to cure other diseases.
Justification: We analyzed over four thousand hospitals, some of which are special hospitals. It is reasonable to simplify and assume that those special hospitals has the ability to cure other diseases.

- Patients of the same disease have similar average state of health across hospitals.

Justification: Large differences in age, sex, etc. will affects the disease-caused mortality rate significantly. Thus reasonably we assume that there is little difference among patients' average health conditions between hospitals. For example, if hospital A treated a 10 year old child, a 30 year old man, and an 80 year old woman, then hospital B shall be assumed as treating a 10 year old girl, 30 year old man, and 80 year old man.

4 Symbols and Definitions

Symbol	Definition
A	Alternatives Comparison Matrix
b	Statewide Percentage of Death
c	Cost
C	Criterion Matrix
d	Disease-Caused 30-day Mortality Rate
d'	Operation-Caused 30-day Mortality Rate
D	Distance Between Patient and Hospital
f	Number of Respondents for each Patient Rating Survey Question Answer
m	Hospital Mortality Rate State Difference
n	Total number of hospitals
p'	Hospital Patients' Rating
p	Hospital Patients' Rating State Difference
q	Total Respondents
r	Hospital Ranking
R	Radius of the Earth
s	Patient Safety State Difference
S_x	State Average Disease-Caused Mortality Rate
S_y	State Average Mortality-Caused Mortality Rate
S_p	State Average Patient Rating
S_s	State Average Patient Safety
S_c	State Average Cost
x	Hospital Disease-Caused Mortality Rate
y	Hospital Operation-Caused Mortality Rate
α	Disease-Caused Mortality Weight
β	Operation-Caused Mortality Weight
κ	Survey Question Weight
ϕ_1	Latitude of Patient
ϕ_2	Latitude of Hospital
ρ_1	Longitude of Patient
ρ_2	Longitude of Hospital

5 Data Collection and Analysis

Since the establishment of our model relied on a large quantity of data, correct interpretation and meaningful analysis is crucial in our modeling process. After initial failed data search and analysis, we found that most of the data we wish to obtain (patient records) from every hospital is unobtainable due to security reasons. Eventually we found and chose the dataset from Medicare.gov and AHRQ [3], obtaining information for a total of 4,806 U.S. hospitals. To account for the different distribution of diseases among states, we took the difference between the hospital's data and the state's average and used it as the crucial criteria for hospital rankings.

6 Comparative Mortality Model

Death is the most tractable outcome of hospital care, since it can be easily measured, is of undisputed importance to everyone, and is very common in hospitals. However, mortality rate itself is a very bad indicator to analyze hospital performance.

The problem is that we cannot know which conditions and diseases can be prevented at a hospital by only looking at its in-hospital mortality rate. For example, two hospitals can have the same mortality rate [4], but hospital A has a high preventable death ratio and a low inevitable death ratio, but hospital B has a much lower preventable death ratio and a high inevitable death ratio. If we only look at the mortality rate, we might conclude that both hospitals are of the same quality, but hospital B is actually better than hospital A since a low preventable death rate means there are less deaths due to preventable (curable) causes.

Inevitable and preventable mortality rates are actually extreme oversimplifications of mortality. In hospitals, there are no inevitable diseases and no absolute preventable mortality. We wish to determine factors that can indirectly signal a hospital's quality without oversimplification.

After analysis, we divided mortality into two categories: disease-caused mortality and operation-caused mortality. We found that many diseased-caused mortality is not a very good indicator of a hospital's overall quality due to uncertainty and locality, but operation-caused mortality actually is. Most highly praised hospitals and famous hospitals in the U.S. have very low operation-caused mortality rates, reflecting that they have the best doctors and equipment.

6.1 Disease-Caused Mortality

There are thousands of diseases and every patient is affected by one or more of them. It is very difficult to calculate the exact accurate mortality for each disease and for each hospital. To get an plausible value, we chose five diseases from the Top 10 Deadliest Diseases List [5]: Acute Myocardial Infarction (AMI) (or Heart Attack), Chronic Obstructive Pulmonary Disease (COPD), Heart Failure (HF), Peumonia (PN), and Stroke (STK).

To account for the fact that some diseases are more common and eminent in some states (such as high heart attack rates in some southern states), we calculated the quotient of the people who died from each disease to the total number of people who died from these diseases.

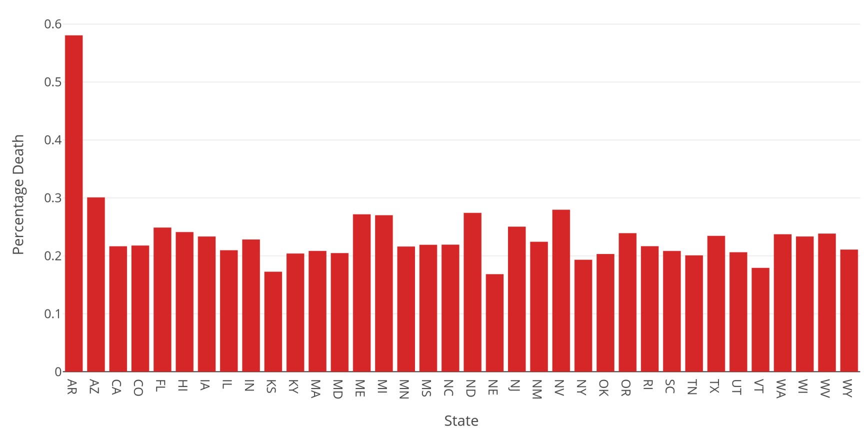


Figure 2: The percentage of heart attack (AMI) deaths in each state

This way, we can give higher weights to states in hospitals where treatment of certain diseases is more important than that disease in another state. To put it simply, we rank the mortality rate according to the "importance factor", which dictates the overall quality of the hospital.
We then multiply this quotient by the mortality rate of each cause of death to get the hospital's disease-caused mortality rate, x:

$$x = \sum_{i=1}^{4}(d_i \cdot b_i)$$

where,
- d_i represents each disease's 30-Day mortality rate,
- b_i represents the statewide percentage of deaths.

Since states vary in death percentages, they also vary on the average level of hospitals. Some medically-advanced states have a higher average hospital quality, while others do not. To solve this problem of variation, we subtracted the state's average from each hospital's disease-caused mortality rate, so that average quality hospitals all have values around 0.

We shall take the sum of the disease-caused mortality rate of all hospitals and divide it by the total number of hospitals (4806), shown by the following formula,

$$S_x = \frac{\sum_{i=1}^{n} x_i}{n}$$

where,
- n represents the number of hospitals in each state.

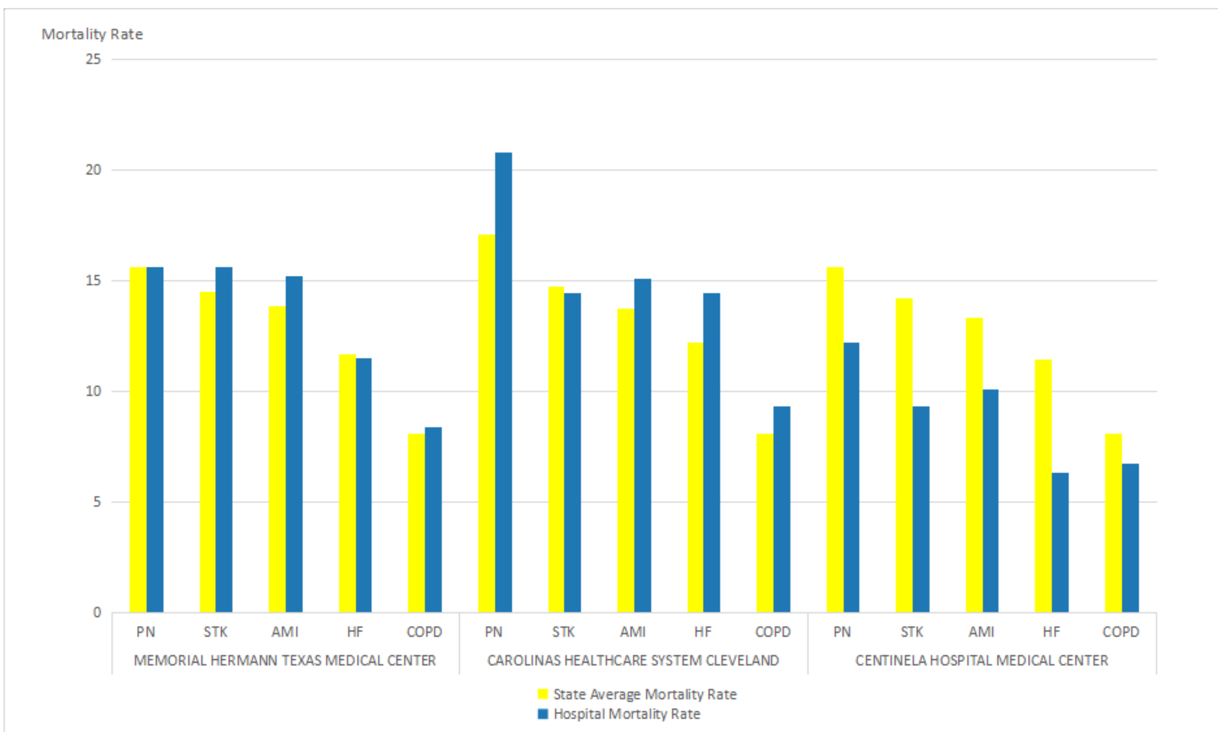


Figure 3: Hospital's Disease-Caused Mortality Rate in Comparison with State Average Mortality Rate

6.2 Operation-Caused Mortality

Operation-Caused Mortality can better reflect a hospital's overall quality. We selected three measure: Deaths among Patients with Serious Treatable Complications after Surgery (PSTCS), Death rate for Coronary Artery Bypass Grafting (CABG), and Serious blood clots after surgery (BC), calculated the hospital operation-caused mortality rate brought by these surgical errors, and compare it to the statewide operation-caused mortality rate.

Similar to the disease-caused mortality rate, we could write the equation of operation-caused mortality, y, as

$$y = \sum_{i=1}^{n} d'_i$$

where,
- d'_i represents the operation-caused mortality rate.

Since these operations do not vary state by state, we will omit the state death percentage from the previous formulas. Adding the hospitals' rates together and taking the average of the sum, we calculated the statewide operation-caused mortality rate, as shown by:

$$S_y = \frac{\sum_{i=1}^{n} y_i}{n}$$

where,
- n represents the number of hospitals in each state.

Taking the difference between each hospital's operation-caused mortality rate and the state average, we can get the operation-caused mortality difference of each hospital.

6.3 Comparative Mortality

Given the two measures, we analyzed the comparative mortality rate of each hospital. However, we can not derive this by simply adding disease-caused mortality and operation-caused mortality, since the latter one plays a more important role of showing the hospital's quality. Also, since we did not apply a weight to the operation-caused mortality rates, it is significantly higher than the disease-caused mortality rates. Therefore, we added weights to them, 4.7 and 3.5 respectively. The comparative mortality, m, can be calculated by the following formula,

$$m = \alpha(x - S_x) + \beta(y - S_y)$$

where,
- α represents disease-caused mortality rate,
- β represents operation-caused mortality rate.

A smaller value means a higher hospital quality and that a larger value means a lower quality.

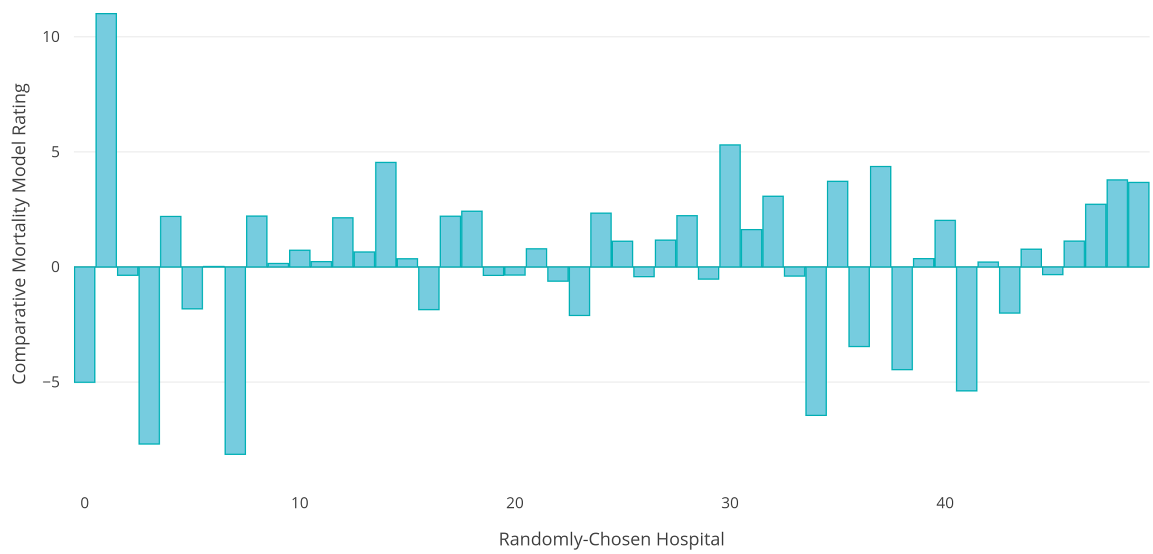


Figure 4: The Comparative Mortality Raring of Random Hospitals

Ranking	Hospital
1	CEDARS-SINAI MEDICAL CENTER
2	CENTINELA HOSPITAL MEDICAL CENTER
3	MAIMONIDES MEDICAL CENTER
4	NEW YORK-PRESBYTERIAN HOSPITAL
5	OROVILLE HOSPITAL
6	HOUSTON METHODIST HOSPITAL
7	NYU HOSPITALS CENTER
8	SOUTHERN CALIFORNIA HOSPITAL AT HOLLYWOOD
9	METHODIST HOSPITAL OF SOUTHERN CA
10	MAIN LINE HOSPITAL LANKENAU

Figure 5: Top 10 Hospitals in the Unites States by Mortality Rate

7 Other Factors

To further assess the quality of hospitals, we introduced three different factors: Patients' Ratings, Patient Safety Indicators, and the average Cost of a major hospital visit.

7.1 Patients' Ratings

To parameterize patient ratings, we analyzed the data from the Hospital Consumer Assessment of Health-Care Providers and Systems (HCAHPS) [6]. The HCAHPS is a questionnaire given to hospital patients asking about the hospital's service during their stay. From this data, we selected ten most important aspects that patients evaluated, and analyzed them.

We chose ten aspects from the questionnaire – room cleanliness, nurse communication, doctor communication, staff responsiveness, discharge information, care transition, quietness at night, and hospital recommendation – measures that can attract or repel patients.

In the questionnaire, most answers are reported in three different categories (always, sometimes, never). For example, in the questionnaire, the patient can answer either the room is always clean, sometimes clean, or never clean. Since these answers are not quantitative, we gave then corresponding values: 5, 3, and 1, respectively. However, some questions only have true/false answers, which we designated corresponding values of 5 and 3.

For each measure, we designated a row matrix of 1x3 (or 1x2) as the number of respondents with each answer (f_i), and a column matrix κ as the answer values of 5, 3, and 1 or 5 . For instance, if 58% of respondents rated the hospitals' rooms as always clean, 32% as sometimes clean, and 10% as never clean, and 1000 people participated in the survey, then

$$f * \kappa = \begin{pmatrix} 1000 * 0.58 \\ 1000 * 0.32 \\ 1000 * 0.10 \end{pmatrix} \begin{pmatrix} 5 & 3 & 1 \end{pmatrix}$$

We can take the sum of this matrix(infinity-norm) and divide by the number of respondents to get a normalized value, then sum all the normalized values for different survey questions, resulting in the equation: Therefore, we apply the formula, shown as

$$p' = \sum_{i=1}^{10} \frac{\|f_i \cdot \kappa_i\|_\infty}{q_i}$$

where,

- f_i represents a the number of respondents for each question choice,
- q_i represents the total number of people who answered the questions of that factor,
- κ_i represents the weight matrix

Since we have ten different factors and each of them will have a score in the range of one to five, then total achievable score is fifty. Each hospital in the United States has its own score, and the higher the score, the better the hospital's service.

As with our comparative mortality model, we will use the difference between each hospital's score and the state average score to better compare hospitals on a national level. Thus,

$$p = (\sum_{i=1}^{10} \frac{\|f_i \cdot \kappa_i\|_\infty}{q_i}) - S_p$$

where $S_p = \frac{\sum_{i=1}^{n} p'_i}{n}$.

Therefore, the higher p is, the better the hospital's patient rankings are.

7.2 Patient Safety Indicators

Patient Safety Indicators (PSIs)[7], developed by the Agency for Healthcare Research and Quality (AHRQ)[3] are a set of measures that measure adverse events that patients might experience in a health care system.

PSIs can measure how well hospitals can keep patients safe after an operation or during an operation, and is mainly affected by hospital quality. A better hospital will usually have lower PSI rates.

PSI 90 (The Patient Safety and Adverse Events Composite)[8] is a measure using ICD-9-CM diagnosis (A type of disease classification)[?] to judge a hospital's overall quality to keep a patient safe. It combines selected PSIs with different weights to provide a clinical judgment.

Component Weight		
PSI	Indicator	Weight
PSI 03	Pressure Ulcer rate	0.059841
PSI 06	Iatrogenic Pneumothorax Rate	0.053497
PSI 08	In-Hospital Fall with Hip Fracture Rate	0.010097
PSI 09	Perioperative Acute Kidney Injury Rate	0.085335
PSI 10	Postoperative Acute Kidney Injury Rate	0.041015
PSI 11	Postoperative Respiratory Failure	0.304936
PSI 12	Perioperative Pulmonary Embolism and Deep Vein Thrombosis Rate Rate	0.208953
PSI 13	Postoperative Sepsis Rate	0.216046
PSI 14	Postoperative Wound Dehiscence Rate	0.013269
PSI 15	Unrecognized Abdominopelvic Accidental Puncture / Laceration Rate	0.007011

[?] Using the information of all American Hospitals, we can calculate the PSI 90 value of each hospital and the statewide rate. Similarly, we took the difference between the hospital rate and the statewide rate, measuring the hospitals' quality of safety.

$$s = PSI90 - S_s = PSI90 - \frac{\sum_{i=1}^{n} PSI90_i}{n}$$

A smaller s value (sometimes negative) means better quality, and vise versa.

7.3 Cost

Since patients' age, condition, length of stay, and healthcare are very different, it is hard to approximate the average amount of money a patient has to pay during a hospital visit. However, using the data available, we found that we can measure the cost by using four costs for different parameters:

$$\text{Heart attack} \tag{1}$$
$$\text{Heart failure} \tag{2}$$
$$\text{Hip or knee replacement} \tag{3}$$
$$\text{Pneumonia} \tag{4}$$

Using these four costs for each hospital, we averaged them and subtracted the state average from these results. This way, we can simplify the payment models at different hospitals using the same parameters.

$$c = \frac{heartAttackCost + heartFailureCost + replacementCost + pneumoniaCost}{4} - S_c$$

8 Ranking Models

To make use of our separate factor models, we need a ranking system that can rank the top U.S. hospitals and local hospitals. To achieve this, we used AHP and our own method, Patient Weighted State Comparison Model (PWSCM) to rank the top 20 American hospitals.

Both the AHP model and our PWSCM model uses a user-input comparison method. For each of the four factors (mortality, patient rating, safety, cost), the user can determine how much each factor means to them, on a scale of 1-10. For example, a user might rate the hospital's mortality as 6, patient ratings as 7, safety as 5, and cost as 4. Therefore, a user-determined importance matrix can be represented as:

$$\begin{bmatrix} 6 & 7 & 5 & 4 \end{bmatrix}$$

Normalizing this matrix, which means dividing each element by the sum of all elements, we get this matrix, which we will call the criterion matrix ($\boldsymbol{C}$):

$$\boldsymbol{C} = \begin{bmatrix} 0.273 & 0.318 & 0.227 & 0.182 \end{bmatrix}$$

8.1 Analytic Hierarchy Process Model

Analytic Hierarchy Process (AHP)[9] is a multi-criteria decision analysis technique developed by Prof. Thomas L. Saaty for organizing and analyzing complex decisions. We will first use this model to assess the rankings of U.S. hospitals.

For the sake of choosing the best hospital (objective), we used mortality rate (m), patient ranking (p), patient safety (s), payment (c) as the criteria, and different hospitals as the alternatives. The hierarchy can be diagrammed as shown below:

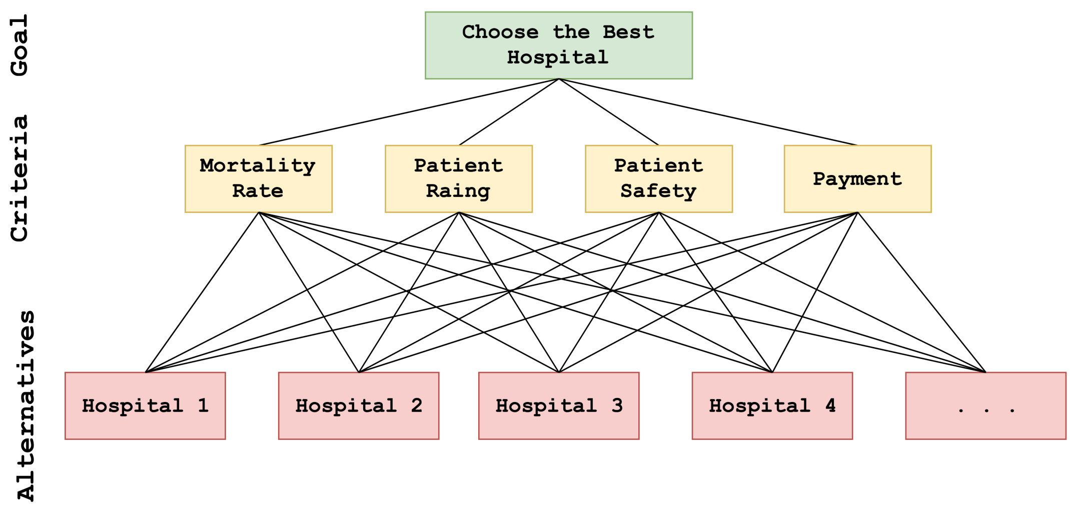


Figure 6: Analytic Hierarchy Process Model

We can set the priorities for each criteria according to the criterion matrix $\boldsymbol{C}$. For our rankings, the criterion matrix we used was:

$$\boldsymbol{C} = \begin{bmatrix} 0.375 & 0.375 & 0.15 & 0.1 \end{bmatrix}$$

We can also see the criterion matrix as a 4x4 matrix with pairwise comparisons, as shown:

$$\boldsymbol{C} = \begin{array}{c} \\ m \\ p \\ s \\ c \end{array} \begin{array}{c} \begin{array}{cccc} m & p & s & c \end{array} \\ \begin{pmatrix} 1 & 1 & 2.5 & 3.75 \\ 1 & 1 & 2.5 & 3.75 \\ 0.4 & 0.4 & 1 & 1.5 \\ \frac{4}{15} & \frac{4}{15} & \frac{2}{3} & 1 \end{pmatrix} \end{array}$$

As we can see, this criterion matrix is consistent, which means that the weights for the criteria are the same as $\boldsymbol{C}$.

Furthermore, we can do a pairwise comparison of all the hospitals in each of the four factors. However, since we have data for 4,806 different hospitals, it would take at least $4 * 4806 * 4806 = 92390544$ computations to get the comparison matrix, then even more to get the principal eigenvectors and eigenvalues. Therefore, we only took the top 200 hospitals ranked by mortality rate to do the AHP ranking.

Using pairwise comparisons, we can determine a comparison matrix $\boldsymbol{A}_k$, with k being each of the four factors. For example, our mortality comparison matrix ($\boldsymbol{A}_1$) is:

$$\begin{pmatrix} 1 & 1.05064 & 1.05071 & \dots & 1.45233 \\ 0.95179 & 1 & 1.00005 & \dots & 1.43821 \\ 0.95173 & 0.99994 & 1 & \dots & 1.36888 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.68855 & 0.69531 & 0.73053 & \dots & 1 \end{pmatrix}$$

Calculating the eigenvalues and eigenvectors of the matrices, we can obtain the normalized principle eigenvalues (primary eigenvectors). This is the priority vector of each hospital to each factor.

Since $\boldsymbol{A}_k$ are consistent matrices, no consistency test is necessary.

We can then evaluate the final ranking by multiplying the hospital's weight and the factor's weight together, the summing the products over the four different factors. This gives us the final rankings of each hospital.

8.2 Patient Weighted State Comparison Model

In our previous AHP model, we chose four criteria for analysis. Though the AHP method gives us relatively accurate and reasonable results, this calculation method is very tedious and has a high computation complexity. The complexity of this algorithm is $O(n^2)$, an undesirable result. In the AHP method, we need to compare hospitals in pairs, normalize their ratings, calculate the primary eigenvalue, and then multiply to obtain their respective proportions. In our AHP analysis, we first ranked the top 200 hospitals based on mortality rankings, then did pair-wise comparisons. Even this simplified algorithm took a long time to complete. With 4806 hospitals as alternatives, and maybe more for different countries or criteria, the AHP method is very ineffective. This method gives us accurate and precise results with the cost of computation complexity.

Therefore, based on the previous AHP model, we made some major improvements and derived our own model, the Patient-Weighted State Comparison Model (PWSCM). The PWSCM model is based on the mortality, patient ratings, safety, and cost, four criteria we determined to be important in evaluating hospital quality. Since the calculation methods for the four criteria are similar but different, we want to maximize some values(such as patient rating, safety) while minimizing other values (mortality, cost). Thus, we can't simply add them together to get a total ranking. We need to analyze them and standardize them based on the same values.

To simplify our calculations, we standardized each rating with a scale from 0-10. We defined the maximum rating, i.e. the best hospital score of each factor a value of 10, and the minimum a value of 0. For factors where we want to minimize the score, we used the following equation:

$$a = -10 * \frac{(score - max)}{max - min}$$

For factors where we want to maximize the score, we used the following equation:

$$a = 10 * \frac{(score - min)}{max - min}$$

Our equation also used user-determined weights, so the priority vector of the factor is our criteria matrix. For our ranking, we also used values of 0.375, 0.375, 0.125, and 0.125. We multiplied these four weights by our reranked score from 0 to 10 obtained for each of the factors, then added the values to obtain a new value, as shown by:

$$r = \sum_{i=1}^{4} \boldsymbol{C}_i * a_i$$

This new numerical score is not very intuitive, so we also standardized this value using equation. We sorted the final values in descending order, and obtained the PWSCM model rankings. We can compare the top 10 hospitals ranked by AHP and PWSCM like so:

Ranking	AHP		PWSCM
1	MAYO CLINIC HOSPITAL		MAYO CLINIC HOSPITAL
2	MAYO CLINIC HOSPITAL ROCHESTER		MAYO CLINIC HOSPITAL ROCHESTER
3	CLEVELAND CLINIC		CITIZENS MEDICAL CENTER
4	CHESTER COUNTY HOSPITAL		CLEVELAND CLINIC
5	SCRIPPS GREEN HOSPITAL		CEDARS-SINAI MEDICAL CENTER
6	SANTA MONICA - UCLA MED CTR & ORTHOPAEDIC HOSPITAL		CHESTER COUNTY HOSPITAL
7	CEDARS-SINAI MEDICAL CENTER		DELAWARE VALLEY HOSPITAL, INC
8	MILLS-PENINSULA MEDICAL CENTER		NEW YORK-PRESBYTERIAN HOSPITAL
9	JOHN T MATHER MEMORIAL HOSPITAL OF PORT JEFFERSON		SANTA YNEZ VALLEY COTTAGE HOSPITAL
10	NEW YORK-PRESBYTERIAN HOSPITAL		MAIN LINE HOSPITAL LANKENAU

Figure 7: Two Models' Top 10 Hospitals Comparison

Most hospitals have no very significant changes in the rankings from the two models, but some hospitals are not correspondent with each other between the two rankings. The reason for this result is that the AHP ranking is not a fully developed ranking. Since we used mortality rates to get 200 hospitals to compare in the AHP model, the result is not comprehensive enough. Some hospitals, such as the CITIZENS MEDICAL CENTER, has a high patient ratings score but a relatively average mortality score. Therefore, it did not even get considered to be part of the AHP rankings, resulting in this difference.

From these comparisons, we can find that the rankings of our PWSCM model is very similar to the rankings of the AHP model, further validating our own model. Also, our model has a low time complexity, O(n), much better than the AHP algorithm. Thereby we can conclude that our model in accurate and efficient multi-criteria decision analysis technique, and we can use it in further analysis, replacing the AHP model.

Each year, US NewsWEBSITE will rank all hospitals in the United States. We selected hospitals ranked TOP20 in the US News 2017-2018 Honor Roll, and compared them with our ratings from AHP and PWSCM.

US News Ranking	Name	PWSCM Ranking	AHP Ranking
1	Mayo Clinic, Rochester, Minnesota	2	2
2	Cleveland Clinic	4	3
3	Johns Hopkins Hospital, Baltimore	72	28
4	Massachusetts General Hospital, Boston	741	#N/A
5	UCSF Medical Center, San Francisco	83	#N/A
6	University of Michigan Hospitals and Health Centers, Ann Arbor	232	74
7	Ronald Reagan UCLA Medical Center, Los Angeles	25	11
8	New York-Presbyterian Hospital, New York	8	10
9	Stanford Health Care-Stanford Hospital, Stanford, California	965	#N/A
10	Hospitals of the University of Pennsylvania-Penn Presbyterian, Philadelphia	39	29
11	Cedars-Sinai Medical Center, Los Angeles	5	7
12	Barnes-Jewish Hospital, St. Louis	537	#N/A
13	Northwestern Memorial Hospital, Chicago	144	64
14	UPMC Presbyterian Shadyside, Pittsburgh	3809	#N/A
15	University of Colorado Hospital, Aurora	944	#N/A
16	Thomas Jefferson University Hospitals, Philadelphia	994	#N/A
17	Duke University Hospital, Durham, North Carolina	1404	#N/A
18	Mount Sinai Hospital, New York	214	71
19	NYU Langone Medical Center, New York	11	13
20	Mayo Clinic Phoenix	1	1

Figure 8: Top 20 Hospitals Ranked By U.S.News in Comparison with Our Models

It can be seen that some rankings obtained by the PWSCM model are not significantly different from those of US News, but there are situations where individual hospital rankings are different. One reason this happens is that the criteria we used were drastically different. US News ranked the hospitals based on the hospital's rankings 16 different specialties, which uses expert opinion (Delphi method) as an important criteria. However, our PWSCM model is mostly based on the perspective of patients. Therefore, the ranking of the PWSCM model is more feasible and accurate for each patient, as expert opinions on hospitals can be very different from patient opinions. Also, some hospitals from the U.S. News ranking were not in the top 200 considered in the AHP model, so they lack rankings in the AHP rankings.

Therefore, we cannot conclude that our model is inaccurate based on U.S. News rankings, but we can see that many hospital rankings were very similar, proving the effectiveness and accuracy of our model to a great extent.

9 Algorithm Diagram

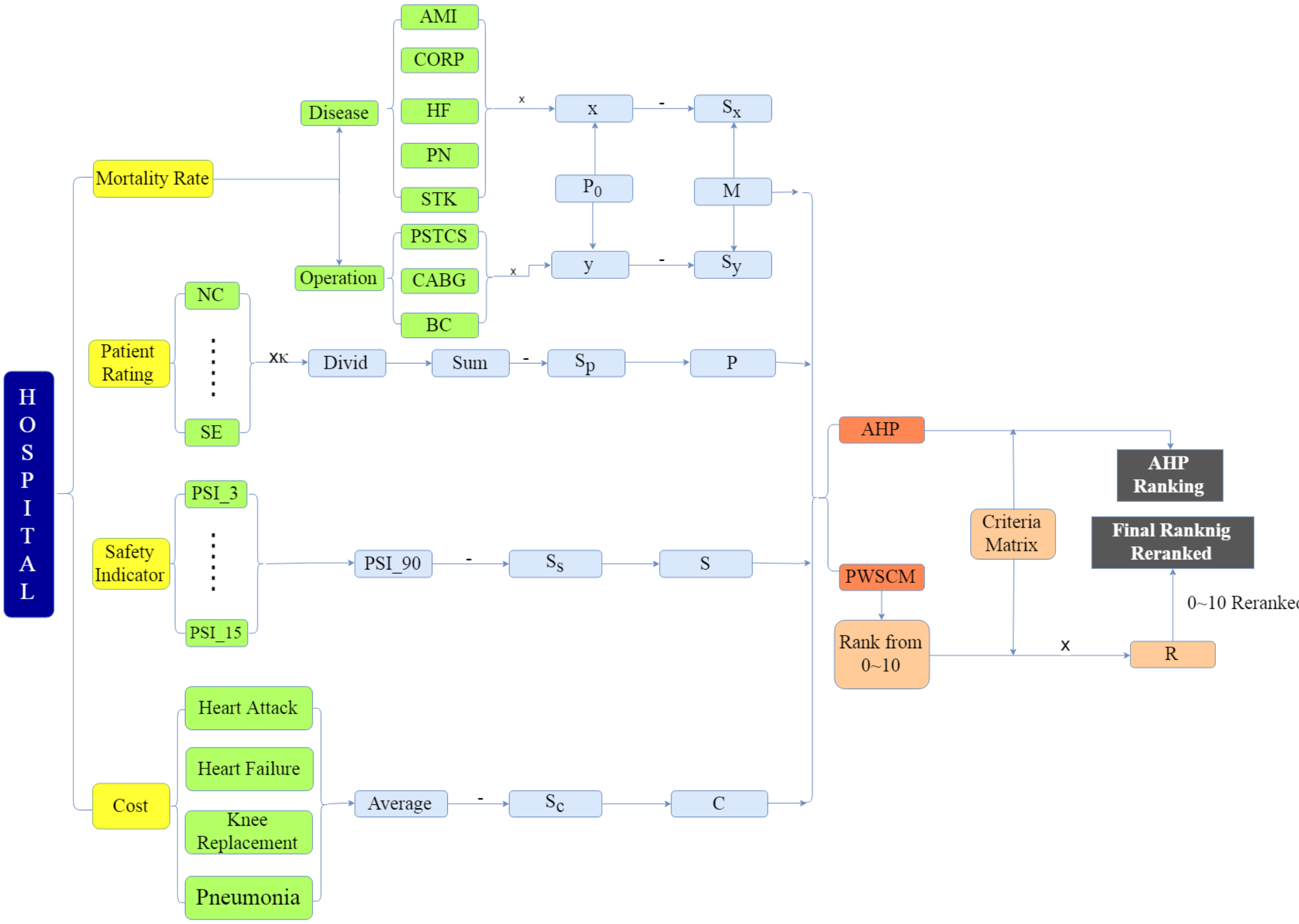


Figure 9: Algorithm Diagram

10 Sensitivity Analysis

For our sensitivity analysis, we adjusted our models so that it did not include the data for patient ratings. Therefore, the hospitals were ranked nationally by their mortality rate, safety, and cost. The following table shows the top 10 hospitals ranked by both methods:

Ranking	AHP	PWSCM
1	CENTINELA HOSPITAL MEDICAL CENTER	CENTINELA HOSPITAL MEDICAL CENTER
2	MAYO CLINIC HOSPITAL	MAYO CLINIC HOSPITAL
3	FAIRVIEW HOSPITAL	CEDARS-SINAI MEDICAL CENTER
4	HOUSTON METHODIST HOSPITAL	FAIRVIEW HOSPITAL
5	EMORY JOHNS CREEK HOSPITAL	HOUSTON METHODIST HOSPITAL
6	MAIN LINE HOSPITAL LANKENAU	OROVILLE HOSPITAL
7	OROVILLE HOSPITAL	NEW YORK-PRESBYTERIAN HOSPITAL
8	NEW YORK-PRESBYTERIAN HOSPITAL	MAIN LINE HOSPITAL LANKENAU
9	CEDARS-SINAI MEDICAL CENTER	EMORY JOHNS CREEK HOSPITAL
10	NYU HOSPITALS CENTER	NYU HOSPITALS CENTER

Figure 10: Top 10 Hospitals Without Patient Rankings

Based on our sensitivity analysis, we can see that both models are very robust against missing parameters and data, and both models' rankings are very similar and sensible, as most of the hospitals on the list ranked top 20 with patient ratings. Therefore, we can conclude that both models are effective against disturbances in data sources.

11 HERS: A Case Study

In order to demonstrate the real-world applications of our model, and to help patients choose a hospital, we created a software, which we called the Hospital Evaluation Ranking System (HERS), that can rank the hospitals near a patient according to the patient's own ranking criteria. To demonstrate this, we chose Los Angeles as a case study. We assume that the patient is located in downtown Los Angeles, using our software to select the best hospital for him based on his condition.

There are a total of 82 hospitals in Los Angeles, and in this case study, we have selected six of them, close to downtown LA, as shown:

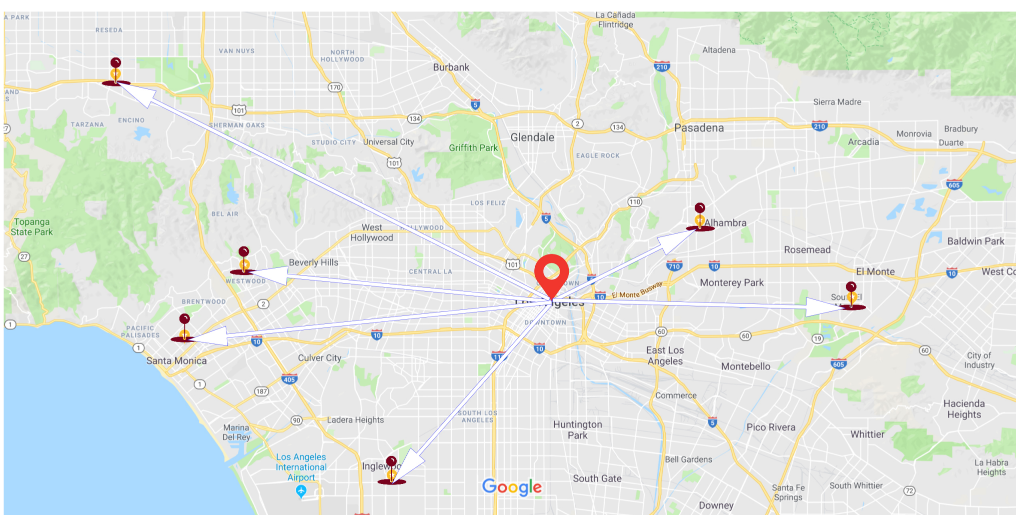


Figure 11: Map of Chosen Hospitals and Patient

Our software asks the patient to input their ZIP code, as each ZIP Code corresponds to a latitude and longitude coordinate. We used the U.S. 2010 Census data to obtain the latitude and longitude of the patient and the hospitals, and employed the Haversine Formula [10] to calculate the distance:

$$D = 2R\arcsin\left(\sqrt{\sin^2\left(\frac{\psi_2-\psi_1}{2}\right)+\cos(\psi_1)\cos(\psi_2)\sin^2\left(\frac{\rho_2-\rho_1}{2}\right)}\right)$$

The parameters ψ and ρ is respectively the latitude and longitude of the patient/hospital.

We assumed that the patient's weights for these four criteria (mortality, patient ratings, patient safety, cost) were 7.5, 7.5, 3, and 2, respectively. Therefore, the criteria matrix $\boldsymbol{C} = \begin{bmatrix}0.375 & 0.375 & 0.15 & 0.1\end{bmatrix}$. Using the PWSCM algorithm, we can rank all national hospitals based on this input. To better visualize the resulting data, we will do a cost-benefit analysis of the distances and the ratings, with distance being the cost and ratings as the benefit. Plotting the graph, we get:

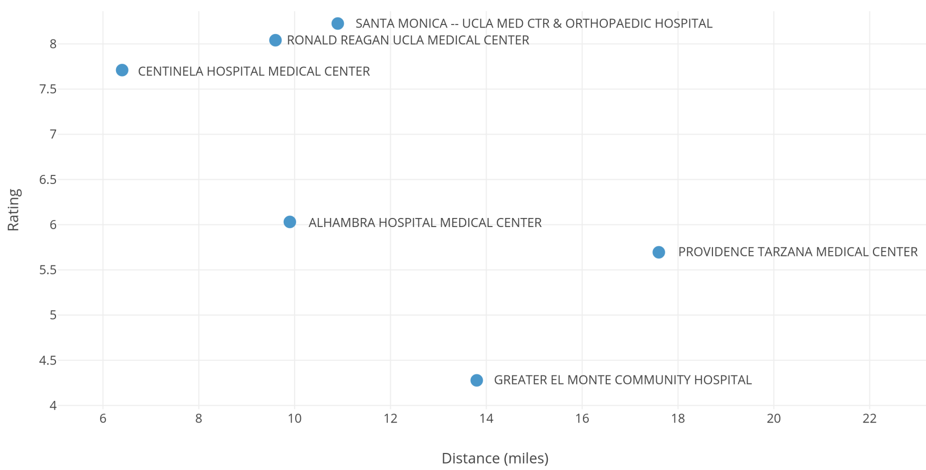


Figure 12: R-D Graph of Chosen Hospitals in LA

We can divide the area in the figure into four small areas as follows:

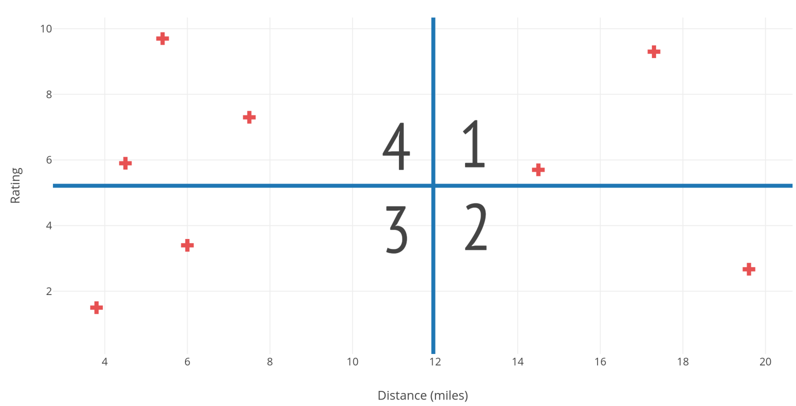


Figure 13: R-D Graph Regions

Normally, if a hospital is in close proximity and has a high score, that is, it is located in the area where "4" is located in the map, the hospital will be the preferred target. However, in real situations, special circumstances may arise.

If a patient with an extreme emergency and needs immediate treatment, hospitals that are located far away (in the "1" and "2" areas) are not good choices. Due to the urgency needed for treatment, we recommend selecting certain hospitals in the "3" and "4" areas according to his more own physical conditions.

If this is a patient suffering from a chronic disease, he may not need to go to the nearest hospital with the closest, but choose a hospital with a better overall assessment. Then hospitals in the "1" and "4" regions will be better choices.

In the case where the distance from the hospital to the patient's location is similar, choosing a hospital with a higher ranking is necessarily a better choice (Figure). If the rankings are comparable, but there is a large difference in distance, we recommend that patients choose a closer hospital.

If the patient during the selection process encounters a situation in which the levels of the two hospitals are significantly different and the distances between the two hospitals are large (Figure: bottom left + top right), they need to consider their own physical condition and decide which to choose.

The choice of the PWSCM model provides great convenience and leaves greater choice-making to the patients themselves. They only need to provide the ZIP Code of their location and their own opinions on factor importance. Our method is more user-friendly than the AHP method, which takes a long time to compute and may waste computer memory.

For the patient in downtown Los Angeles that was introduced in the case study, an analysis of graphs shows that under normal circumstances, choosing one of the three hospitals located in the "4" region of the graph would be a good choice, either: Centinela Hospital Medical Center, Ronald Reagan UCLA Medical Center, and Santa Monica UCLA Med CTR and Orthopaedic Hospital.

12 Strengths and Weaknesses

12.1 Strengths

1 Our model is very adaptive and it can be used in different countries.

- We only used data from the United States, but if data for the specific measures we utilized in our model were available for other countries, it still can be used for that country.

2 Our model places an emphasis on patients' experiences, so our model more suitable for patients seeking healthcare, who might consider comfort as an important measure.

- In our analysis, we gave patient experiences a weight of 0.35, and this can be set by the patient themselves, either higher or lower.

3 Our model can be highly customized for patients.

- Most of our parameters can be set by patients themselves, so our model and our rankings presented in this paper is not a set one; rather, every patient can have his/her own different hospital rankings based on their preferences.

4 We built two different ranking models (AHPM and PWSCM) and compared them to the U.S. News 2017-18 Best Hospitals Honor Roll, and only a few differences occurred.

- We compared our model with an authoritative ranking, and found that many actually corresponded with the authoritative rankings. This further proved the accuracy and reliability of our model.

5 Our model is very user friendly.

- We have developed a software that has a graphical interface, and only presents easy to understand information for patients who have none or almost none mathematical knowledge.

6 We have considered different situations that the patient can be in, and gave suggestions for most of them.
- Using the distance-ranking graph, patients can choose their own hospital based on their condition and situation.

7 Compared with an AHP algorithm, our PWSCM is faster, efficient, and accurate.

12.2 Weaknesses

1 We only considered five diseases and three operations for our mortality model and did not subdivide them into detailed and specific diseases.
- Since there are a lot of different diseases patients might have, we cannot consider all of them in one model. So we only selected 5 disease mortality rates to represent the overall quality of a hospital. This might result in partial evaluation of a hospital's medical quality.

2 We applied the state's average data to hospitals without data in that measure, and it would probably affect the final rankings.
- For some smaller hospitals, we do not have complete information, so we substituted the state average data into these hospitals. This might result in a deviation of final rankings.

3 We did not fully consider every aspect of a hospital, such as the scale, staff assignment, and number of beds.
- Since we did not include these measures, our final rankings might not be very accurate.

4 In our models, we did not consider hospitals with specialties in one or more disease or operations.
- Since we do not have complete information, we can only assess the quality of hospitals in some mortality measures. However, some hospitals great in one area might not have a very good overall medical quality.

5 In our models, we defined a lower cost as being better.
- However, cost should not be considered linearly. Some hospitals provide great service whilst having a relatively high cost. Our models associated cost independently, so cost might not be the best measure to assess the quality of a hospital.

6 We did not analyze expert opinion in our models.

13 Conclusion and Future Work

Our goal was to devise a mathematical model to assess the quality of a hospital using mortality rates, then other factors. We built three models to determine a hospital's quality based on an United States hospital database. Our models are the CMM (Comparative Mortality Model), the AHP model, and our PWSCM (Patient-Weighted State Comparison Model)

In our CMM, we fully considered mortality rate as consisting of two simplified factors: Inevitable mortality and preventable mortality, then projected these ideas onto given data, generating the disease-caused mortality model and operation-caused mortality model. Using different weights and state comparisons to combine these two factors, we can reflect a hospital's actual medical quality.

However, to provide a full evaluation, other factors need to be included. We used patient ratings, patient safety, and cost as other metrics to evaluate a hospital. These were derived similar to the CMM method.

After getting the comparisons between hospitals and state averages, we utilized AHP to rank hospitals nationally. However, AHP is too complicated to do pairwise hospital comparisons over four thousand hospitals, so we did a partial ranking of the top 200 hospitals by mortality. To improve the AHP model and avoid its complexity, we developed PWSCM. This method assigns each hospital's state-wise comparison a value ranging from 1-10, then applied the weights. Both evaluation and ranking models ranked MAYO CLINIC HOSPITAL as the best hospital.

To utilize our models in a real-world situation, we also developed a software system called HERS, which uses user-input importance measures of the four factors, then analyzed the best hospitals closest to the patient's ZIP code. The software system generates a distance-ranking graph, which the patient can use to choose the best hospital for their circumstances.

Given more time, we can add more criteria that can affect the hospital quality, such as preciser disease classification, staff assignment, hospital scale, etc. We can also rank the best hospitals by specialties which corresponds to the patient's condition.

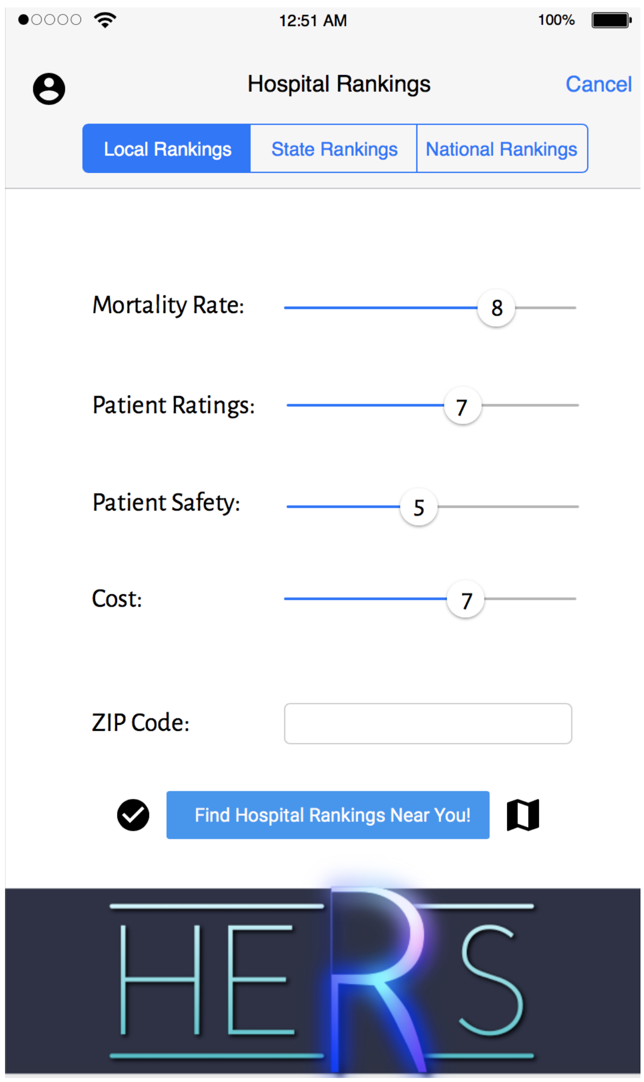


Figure 14: HERS Software

14 Appendix

14.1 Top 10 Leading Causes of Death in the United States

Rank	Disease
1	Ischemic Heart Disease or Coronary Artery Disease
2	Stroke
3	Lower Respiratory Infections
4	Chronic Obstructive Pulmonary Disease
5	Trachea, Bronchus, and Lung Cancers
6	Diabetes Mellitus
7	Alzheimer's Disease and Other Dementias
8	Dehydration due to Diarrheal Diseases
9	Tuberculosis
10	Cirrhosis

14.2 Code

AHPRanking.m

```
numRows = height(AHP);
comp_matrix = [1, 1, 5/2, 15/4; 1, 1, 5/2, 15/4; 2/5, 2/5, 1, 3/2; 4/15, 4/15, 2/3, 1];
AHP1 = AHP(:,[7:10,1]);
mort_matrix = zeros(numRows, numRows);
patient_matrix = zeros(numRows, numRows);
psi_matrix = zeros(numRows, numRows);
spending_matrix = zeros(numRows, numRows);
for i=1:numRows
    for j = 1:numRows
        mort_matrix(i,j) = AHP1{i,1}/AHP1{j,1};
        patient_matrix(i,j) = AHP1{i,2}/AHP1{j,2};
        psi_matrix(i,j) = AHP1{i,3}/AHP1{j,3};
        spending_matrix(i,j) = AHP1{i,4}/AHP1{j,4};
    end
end
[mort_Weight, ~] = eig(mort_matrix);
[patient_Weight, ~] = eig(patient_matrix);
[psi_Weight, ~] = eig(psi_matrix);
[spending_Weight, ~] = eig(spending_matrix);
mort_Weight = mort_Weight(:,1);
patient_Weight = patient_Weight(:,1);
psi_Weight = psi_Weight(:,1);
spending_Weight = spending_Weight(:,1);
mort_Weight = mort_Weight./sum(mort_Weight);
patient_Weight = patient_Weight./sum(patient_Weight);
psi_Weight = psi_Weight./sum(psi_Weight);
spending_Weight = spending_Weight./sum(spending_Weight);
for i = 1:numRows
    AHP1{i,6} = 0.375*mort_Weight(i,1)+0.375*patient_Weight(i,1)+0.15*psi_Weight(i,1)+0.1*
    spending_Weight(i,1);
end
```

HospitalDataAnalysis.py

```

import pandas as pd
import math
import numpy as np
df = pd.read_csv('../Data/Hospital_Revised_Flatfiles/Deaths.csv')
df = df.drop(['Provider ID', 'Address', 'City', 'ZIP Code', 'County Name', 'Phone Number', '
    Compared to National', 'Lower Estimate', 'Higher Estimate','Footnote', 'Measure Start
    Date', 'Measure End Date'], axis = 1)
df['Ranking'] = 0
```

```
pdf = pd.read_csv('../Data/Hospital_Mortality.csv')
for row in df.itertuples():
    for row2 in pdf.itertuples():
        if row[2] == row2[1] and row[3] == row2[2]:
            if row[6] != 'Not Available':
                df.loc[row.Index, 'Ranking'] = float(row[6])*float(row2[9])
            else:
                if row[3] == 'Acute Myocardial Infarction (AMI) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = 13.6*float(row2[9])
                elif row[3] == 'Death rate for chronic obstructive pulmonary disease (COPD) patients':
                    df.loc[row.Index, 'Ranking'] = 8*float(row2[9])
                elif row[3] == 'Death rate for stroke patients':
                    df.loc[row.Index, 'Ranking'] = 14.6*float(row2[9])
                elif row[3] == 'Heart failure (HF) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = 11.9*float(row2[9])
                elif row[3] == 'Pneumonia (PN) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = 15.9*float(row2[9])
        elif row[2] not in row2 and row[3] == row2[2]:
            if row[6] != 'Not Available':
                if row[3] == 'Acute Myocardial Infarction (AMI) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = float(row[6])*0.23435
                elif row[3] == 'Death rate for chronic obstructive pulmonary disease (COPD) patients':
                    df.loc[row.Index, 'Ranking'] = float(row[6])*0.05117
                elif row[3] == 'Death rate for stroke patients':
                    df.loc[row.Index, 'Ranking'] = float(row[6])*0.33798
                elif row[3] == 'Heart failure (HF) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = float(row[6])*0.19212
                elif row[3] == 'Pneumonia (PN) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = float(row[6])*0.18437
            else:
                if row[3] == 'Acute Myocardial Infarction (AMI) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = 13.6*0.23435
                elif row[3] == 'Death rate for chronic obstructive pulmonary disease (COPD) patients':
                    df.loc[row.Index, 'Ranking'] = 8*0.05117
                elif row[3] == 'Death rate for stroke patients':
                    df.loc[row.Index, 'Ranking'] = 14.6*0.33798
                elif row[3] == 'Heart failure (HF) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = 11.9*0.19212
                elif row[3] == 'Pneumonia (PN) 30-Day Mortality Rate':
                    df.loc[row.Index, 'Ranking'] = 15.9*0.18437
ef = pd.read_csv('1_Prime.csv')
ef = ef.drop(['Unnamed: 0', 'Provider ID', 'Address', 'City', 'ZIP Code', 'County Name', 'Phone Number', 'Unnamed: 16', 'Unnamed: 22', 'Unnamed: 23', 'Unnamed: 24' ], axis = 1)
ef = ef.drop(['Compared to National', 'Unnamed: 13', 'Lower Estimate', 'Higher Estimate', 'Footnote', 'Measure Start Date', 'Measure End Date'], axis = 1)
ef = ef.replace(np.nan, 'Not Available', regex=True)
for row in ef.itertuples():
    if row[7] != 'Not Available':
        ef.loc[row.Index, 'Score'] = row[7]
ef = ef.drop('Unnamed: 15', axis = 1)
df.loc[:, 'Score'] = ef.loc[:, 'Score']
for row in df.itertuples():
    if row[6] != 'Not Available':
        if row[3] == 'Death rate for CABG':
            df.loc[row.Index, 'Ranking'] = float(row[6])*0.4
        if row[3] == 'Serious blood clots after surgery':
            df.loc[row.Index, 'Ranking'] = float(row[6])*0.35
        if row[3] == 'Deaths among Patients with Serious Treatable Complications after Surgery':
            df.loc[row.Index, 'Ranking'] = float(row[6])*0.025
    else:
        if row[3] == 'Death rate for CABG':
            df.loc[row.Index, 'Ranking'] = 3.24*0.4
        if row[3] == 'Serious blood clots after surgery':
            df.loc[row.Index, 'Ranking'] = 4.35*0.35
        if row[3] == 'Deaths among Patients with Serious Treatable Complications after
```

```
    Surgery':
df['Category'] = hosList
df.to_csv('HosCateData.csv')
gf = df.pivot_table(values='Ranking', index=['State', 'Hospital Name'], columns=['Category'],
     aggfunc=np.sum)
hf = df.set_index('State')
```

Rankings.py

```

import pandas as pd
import numpy as np
import matplotlib as plt
df = pd.read_csv('../../Dat/HospitalInformation.csv')
df.sort_values('P', ascending = False)
df['MortalityRanked'] = 0
df['PatientRanked'] = 0
df['PSIRanked'] = 0
df['SpendingRanked'] = 0
l_mort_min = df['Mortality'].min()
l_mort_max = df['Mortality'].max()
l_mort_range = l_mort_max - l_mort_min
l_patient_min = df['P'].min()
l_patient_max = df['P'].max()
l_patient_range = l_patient_max - l_patient_min
l_psi_min = df['PSI'].min()
l_psi_max = df['PSI'].max()
l_psi_range = l_psi_max - l_psi_min
l_spending_min = df['Spending'].min()
l_spending_max = df['Spending'].max()
l_spending_range = l_spending_max - l_spending_min
for row in df.itertuples():
    df.loc[row.Index, 'MortalityRanked'] = 10*(float(row[7]) - l_mort_min)/l_mort_range
    df.loc[row.Index, 'PatientRanked'] = 10*(float(row[8]) - l_patient_min)/l_patient_range
    df.loc[row.Index, 'PSIRanked'] = -10*(float(row[9]) - l_psi_max)/l_psi_range
    df.loc[row.Index, 'SpendingRanked'] = -10*(float(row[10]) - l_spending_max)/
    l_spending_range
gf = df.drop(['Mortality', 'P', 'PSI', 'Spending'], axis = 1)
gf.to_csv('RankedHospitalInfo.csv')
gf['TotalRanking'] = 0
for row in gf.itertuples():
    gf.loc[row.Index, 'TotalRanking'] = row[7]*0.375 + row[9]*0.15 + row[10]*0.10
gf['TotalRankingReranked'] = 0
l_min = gf['TotalRanking'].min()
l_max = gf['TotalRanking'].max()
l_range = l_max - l_min
for row in gf.itertuples():
    gf.loc[row.Index, 'TotalRankingReranked'] = 10*(float(row[11]) - l_min)/l_range
gf.sort_values(by='TotalRankingReranked', ascending = False)
gf.to_csv('FinalRankingsSens.csv')
```

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