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2018
MCM/ICM
Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Summary

"Health strategies are the same as wealth strategies. They are designed to last a lifetime"

----Natasha Deonarain, MD, MBA [35]

Healthcare is one of the most important aspects in a person's life. Many are simply content with a long and healthy life. To maintain a long and healthy life, an outstanding hospital with capable doctors are needed. To determine the quality of a hospital, our paper proposes a two-part calculation.

The first part is of mortality rates, but of course, we cannot simply look at the death rates as it does not give an accurate evaluation of a hospital's quality. Evitable and inevitable death rates are fundamental to figure out the doctors' capability. We determined three most important factors that affect evitable and inevitable deaths, they are diseases, accidents, and resource shortages. Those who die due to these factors are then separated into four age groups for further analysis. After further steps, we can find the percentage of people who should be but not saved out of the total number of patients who should survive. This allows us to tell the quality of each hospital by the evitable death rate.

Part two is made up of the other factors that influence a hospital's quality, which are the ratio of doctors to patients, capacity, equipment, and efficiency. The entropy weighting method is used to determine the top five hospitals without considering the influence of mortality. After further analysis using the Analytic Hierarchy Process (AHP), the best hospital is picked considering all five factors. Both methods quantify the quality of a hospital and endow the criteria and/or alternatives with weights.

Ultimately, our calculations will determine the best hospital out of fifty hospitals by mortality, ratio, capacity, equipment and efficiency.

Content

1. Introduction

1.1 Problem Review

Healthcare is indisputably one of the most important factors in living a long, healthy life. Usually, people would have a choice in where they would seek treatment — especially in a non-emergency situation — and they would pick the best hospital available. There are many points in picking hospitals to take into consideration, with the most important few being mortality, which is impacted by diseases, accidents, resource shortages and so on, and other factors such as capacity and equipment. Based on the factors mentioned, we can tell the overall quality of a hospital and therefore be able to pick the best place to seek treatment.

1.2 Preliminary Analysis

To evaluate the quality of different hospitals, the first step is to pinpoint factors that affect it and the level of the importance for each of them. The most crucial elements include mortality, the doctor-to-patient ratio, capacity, expense on equipment and efficiency.

The total number of death cases in a hospital is not a good measurement of its quality, instead, the percentage of people who can be saved but were not due to various reasons, such as doctor errors, can be used for comparison. According to the relative degree of changes of each element, the weights of the different factors' impact on the overall result can be determined by using the entropy method, which is suitable for analyzing a large sample of 50 hospitals. Hence, the top five hospitals can be chosen to be further analyzed using the AHP method with consideration of mortality and other factors. A reasonable ranking of the hospitals will then be generated using mathematics.

2. Assumptions and Symbols

2.1 Assumptions and Justifications

1. **Assumption: The data used in our model are not collected from real life hospitals but randomly generated using Excel.**
   Justification: Most of the data we need is not made available to the public and there is not a pre-existing database provided for us to work with.

2. **Assumption: The data randomly made up by using excel will be similar to the real-life data**
   Justification: We will make up the data with regards to the real-life data, then

we will be using excel to generate random numbers in a range we made based on the real numbers.

3. **Assumption: Models only consider the impacts of the factors discussed above.** [2]
   Justification: There are innumerable factors that directly or indirectly influence the quality of a hospital. It is impossible to cover them all, and thus, only the most important ones should be included.

4. **Assumption: The factors do not influence each other.** [2]
   Justification: A good hospital usually has larger capacity, more advanced equipment etc. and vice versa for a bad hospital. These factors are all related and it would be easier to distinguish the higher-level hospitals at a glance of the data. As a result, our team is building a math model that analyzes the quality of the hospitals while ignoring the relationships between each factor. It is overcome by independently generating data for each factor in Excel.

5. **Assumption: We are only including data from general hospitals; no children's hospitals are considered.**
   Justification: The patients that go to children's hospitals are mostly at a young age. It will cause a significant increase in the number of young patients in that hospital, which is a special case compared to the general hospitals. Therefore, children's hospitals are not well representative samples in our model.

6. **Assumption: We assume the data of other factors of 50 hospitals all fit in the range we set.**
   Justification: The range is based on hospitals' websites, their annual report and social surveys, including both big, famous hospitals and small, local hospitals, which can, to a great extent, represent 50 different hospitals.

2.2 Symbols

A1	The percentage of people in Age group 1, <5	**M%**	Actual Mortality rate in each hospital
A2	The percentage of people in Age group 2, 5~40	**EM%**	Expected death rate (`inevitable deaths)
A3	The percentage of people in Age group 3, 40~65	**C1**	Ratio of doctor to patients
A4	The percentage of people in Age group 4, >65	**C2**	Capacity
N1	Inevitable death rate for age group 1	**C3**	Expenses on equipments
N2	Inevitable death rate for age group 2	**C4**	Efficiency (Average waiting time)
N3	Inevitable death rate for age group 3	**H1, H2…H50**	Hospital 1 to 50
N4	Inevitable death rate for age group 4	**T**	Total number of patients in a hospital
R1	Total percent of patients died of inevitable diseases	**m**	the percentage of people who should be but not saved out of the total number of patients who should survive
R2	Total percent of patients died of inevitable serious injuries		
R3	Total percent of patients died of shortage of resources		

3. Mortality

When people are looking for hospitals, mortality is one of the most intuitive factors. Since large hospitals usually have a larger number of deaths due to the number of patients that go there, using just the total number of deaths to judge the hospitals may not be a good measure of the quality of the hospitals. However, expressing the outcomes as percentages will make the quality of the hospitals comparable. Mortality is separated into two kinds – evitable and inevitable deaths. Evitable deaths are the ones that could and should have been avoided. They are mostly caused by the mistakes of the hospital. A high evitable death count is a sign that illustrates the poor quality of a hospital. Therefore, the evitable death rates will be very good telling signs of whether a hospital is good or not.

3.1 Research and Data Collection

While cases of inevitable deaths are relatively rare, there are innumerable factors that need to be taken into consideration to determine evitable deaths. As a result, our team calculated the inevitable death rate and used the difference between the actual mortality rate and the inevitable death rate to determine the evitable death rate. The biggest causes of inevitable deaths are diseases, accidents, and resource shortages. As people of different ages have different inevitable death rates, we have also separated the people who have died from the above factors into four groups in terms of their age. The four age groups are <5, 5~40, 40~65 and >65. These age groups are determined by their similar death rates in each of the three categories (disease, accidents, and resource shortages) based on the data found on the internet. [1]

3.1.1 Ranges

By using the limited data, we have found that is available, we set a range using the lowest percentage to be the minimum and the highest percentage to be the maximum, assuming the 50 hospitals fit in these ranges. Excel is the software used to make up the data by putting in the range in equation (1): [8] [20] [21] [22] [23] [24]

$$\text{Random } \% = \text{min } \% + \text{RAND}() \times (\text{max } \% - \text{min } \%) \tag{1}$$

Symbol	Range (out of T)	Symbol	Range (out of T)
A1	11%-20%	M%	3.5%-5.0%
A2	13%-34%	R1	1.5%-2.6%
A3	16%-38%	R2	0.6%-1.0%
A4	1-(A1+A2+A3)	R3	0.08%-0.1%

Figure1. Range of Symbols

3.2 Calculation

3.2.1 Percentage calculation

Referring to the real-life data from previous research, we adjusted the unit to per hospital per month, shown in **Figure 2**. [8] [9] [10] [11] [12] [13] [14] [17] [19]

Symbol	Disease	Accident	Resources Shortage
G1(Age<5)	0.44	0.4485	0.01183
G2(Age 5-39)	2.237	4.155627	0.2087
G3(Age 40-65)	22.631	1.2153	1.477
G4(Age 65+)	145.589	0.019969	0.4934
Tol (total number of patients)	171	5.83956	2.193

Figure 2. Real-life Data Table

In this equation,

$$\frac{\frac{G1}{Tot}\times R1\times T}{T\times A1} \tag{2}$$

the numerator calculates the number of patients in Group 1 who died of diseases, and the denominator calculates the total number of patients in G1. The result of the equation is the percentage of patients in G1 died of disease. The same logic is applied in the process of calculating the percentage of people from different age groups who died of accidents and resources shortage in each hospital.

3.2.2 Calculation for ~~m~~

Since each hospital is randomly given a different percentage of patients in each age group within ranges, N1 is calculated to be the average percentage of the fifty hospitals.

	Diseases	Accidents	Shortage	SUM
A1	0.00033487	4.02E-03	3.11E-05	N1=0.004386643
A2	0.00117636	2.54E-02	3.74E-04	N2=0.026986606
A3	0.009513307	6.02E-03	2.13E-03	N3=0.0031373249
A4	0.05753827	9.36E-05	6.82E-04	N4=0.058314188

Figure 3. Calculations of N1-N4

The total inevitable death rate—EM% in each hospital was calculated by adding up the products of the percentage of patients in an age group (An) and the corresponding percentage of the total inevitable death rate of that age group (Nn), shown in the equation (3).

$$EM\% = A1\times N1+A2\times N2+A3\times N3+A4\times N4 \tag{3}$$

The equation for ~~m~~:

$$\text{m} = \frac{M\%-EM\%}{1-EM\%} \tag{4}$$

calculates the percentage of people who should be but not saved out of the total number of patients who should survive. The lower the ~~m~~, the better the quality of the hospital. The ranking of the hospitals based on ~~m~~ is shown in **Figure 4.**

Hospital name	~~m~~
H6	-0.001353409
H23	-0.000930876
H1	0.001758726
H29	0.002136629
H5	0.002233788
H11	0.002424653
H45	0.002438193
H40	0.003020473
H30	0.003478367
H17	0.003546641
H13	0.004008123
…………	
H24	0.018109298
H7	0.018696143
H26	0.019389289
H32	0.020232196
H22	0.02106048
H39	0.023345556
H42	0.02614771

Figure 4. ~~m~~ Result Ranking

4. Other Factors

In addition to mortality, there are other factors that one might want to use in measuring the overall quality of a hospital. A few possible variables include:

- Ratio of doctors to patients in the hospital
- Capacity - the number of ward beds available in the hospital
- Expense - the amount of money a hospital spends on medical devices maintenance and updating new equipment per year
- Efficiency - the average time a patient needs to wait before seeing a doctor

4.1 Research and Data collection

Similar to the method we used for mortality, the ranges for these four factors were calculated in the same way. For the first three factors, the ratio of doctors to patients [4], capacity and expenditures are similar in the aspect that the higher the value, the better the quality of the hospital. Whereas it is the opposite for the waiting time. Since the data needs to be consistent, some adjustments were made to the data. The range of the waiting time is from 2.5 to 3.5 hours, so we took 3.5 and subtracted it by each value in the C4 column. Based on this, the data was generated as shown in **Figure 5**.

	C1	C2	C3	C4
H1	0.001957	259	7.81	0.78
H2	0.00277	452	7.38	0.75
H3	0.002967	295	7.87	0.33
H4	0.002062	347	7.26	0.91
H5	0.001866	374	6.61	0.34
H6	0.0025	495	5.91	0.96
H7	0.001905	338	6.82	0.79
H8	0.002012	203	6.29	0.8
H9	0.003049	346	7.22	0.55
H10	0.001739	322	6.72	0.55
H11	0.002242	498	7.8	0.76
H12	0.003012	429	6.61	0.29
H13	0.002688	243	8.25	0.73
H14	0.001938	354	6.75	0.72
H15	0.002331	371	7.08	0.34
H16	0.001706	405	6.03	0.65
H17	0.002237	412	6.31	0.31
H18	0.002128	489	5.96	0.18
H19	0.002033	233	8.03	0.09
H20	0.002985	383	8.41	0.28
H21	0.00177	410	7.94	0.15
H22	0.001916	456	6.01	0.31
H23	0.001712	483	7.45	0.85
H24	0.001805	268	7.41	0.13
H25	0.002584	459	7.83	0.53
H26	0.003115	227	6.08	0.4
H26	0.003115	227	6.08	0.4
H27	0.002062	404	6.2	0.23
H28	0.00198	392	5.71	0.83
H29	0.002985	293	7.64	0.99
H30	0.002075	491	8.19	0.74
H31	0.002703	398	8.1	0.75
H32	0.002232	270	6.27	0.44
H33	0.001761	332	7.92	0.19
H34	0.00241	341	7.55	0.29
H35	0.001835	209	7.38	0.34
H36	0.002924	478	7.2	0.98
H37	0.002717	215	6.96	0.11
H38	0.002755	205	6.83	0.29
H39	0.002227	329	6.45	0.32
H40	0.002358	448	5.62	0.95
H41	0.001988	329	6.93	0.18
H42	0.001718	401	7.57	0.64
H43	0.001779	215	5.63	0.9
H44	0.002342	215	7.14	0.64
H45	0.002392	355	6.23	0.29
H46	0.001709	426	6.57	0.62
H47	0.003145	403	6.5	0.93
H48	0.001866	216	6.34	0.19
H49	0.00303	362	5.51	0.25
H50	0.001795	288	6.22	0.32
sum	0.113817	17566	346.5	25.89

Figure 5. Research and Database

Factor	Range (monthly)	Factor	Range (Monthly)
C1	1:300-1:600	C2	200-500
C3	5.5-8.5(million)	C4	2.5-3.5(hours)

Figure 6. Range of Symbols [3] [4] [5] [6]

4.2 Calculation

As an objective weight method, the entropy weighting method fully considers the information provided by the evaluation index, and therefore, has very high reliability and precision. [26] Although it lacks horizontal comparison between each factor, the objective of using this, which is to eliminate 45 hospitals and leave only the best five, is sufficiently achieved. The AHP method is advantageous for its systematicity and conciseness, [27] and its major weakness is overcome by using the weights got from 4.2.1 and data from 4.1 to objectively rather than subjectively determine the degree of preference and pick the best hospital.

4.2.1 Entropy Method

Use the data to create a 50×4 matrix like the general one illustrated in **Figure 7.** The columns C1 to C4 represent the four factors that affect the quality of hospitals, while the rows H1 to H50 represent 50 different hospitals.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{12} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

Figure 7. Matrix 50×4 [29]

According to the formula (5), the sum of the 50 numbers in each column was calculated and each entry (x_{ij}) was divided by its corresponding column sum to form a new matrix P_{ij}, as shown in **Figure 8**. This indicates the weight of index of hospital i under the factor j. [7] [29]

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad (5) \qquad\qquad e_j = -k \sum_{i=1}^{m} p_{ij} \cdot \ln p_{ij} \quad (6)$$

	C1	C2	C3	C4		C1	C2	C3	C4
H1	0.017194268	0.014744	0.022539683	0.030127462	H26	0.027368495	0.012923	0.017546898	0.015449981
H2	0.024337313	0.025732	0.021298701	0.028968714	H27	0.018116802	0.022999	0.017893218	0.008883739
H3	0.026068162	0.016794	0.022712843	0.012746234	H28	0.017396347	0.022316	0.016479076	0.03205871
H4	0.018116802	0.019754	0.020952381	0.035148706	H29	0.026226311	0.01668	0.022049062	0.038238702
H5	0.016394739	0.021291	0.019076479	0.013132484	H30	0.01823102	0.027952	0.023636364	0.028582464
H6	0.021965084	0.028179	0.017056277	0.037079954	H31	0.023748649	0.022657	0.023376623	0.028968714
H7	0.016737394	0.019242	0.01968254	0.030513712	H32	0.019610427	0.015371	0.018095238	0.016994979
H8	0.0176775	0.011556	0.018152958	0.030899961	H33	0.015472205	0.0189	0.022857143	0.007338741
H9	0.026788617	0.019697	0.020836941	0.021243723	H34	0.021174341	0.019413	0.021789322	0.011201236
H10	0.015278913	0.018331	0.019393939	0.021243723	H35	0.016122372	0.011898	0.021298701	0.013132484
H11	0.019698288	0.02835	0.022510823	0.029354963	H36	0.025690363	0.027212	0.020779221	0.037852453
H12	0.026463534	0.024422	0.019076479	0.011201236	H37	0.023871654	0.01224	0.02008658	0.004248745
H13	0.023616859	0.013834	0.023809524	0.028196215	H38	0.024205523	0.01167	0.0197114	0.011201236
H14	0.017027333	0.020153	0.019480519	0.027809965	H39	0.019566497	0.018729	0.018614719	0.012359985
H15	0.020480245	0.02112	0.0204329	0.013132484	H40	0.020717468	0.025504	0.016219336	0.036693704
H16	0.014988974	0.023056	0.017402597	0.025106219	H41	0.017466635	0.018729	0.02	0.006952491
H17	0.019654357	0.023454	0.018210678	0.011973735	H42	0.015094406	0.022828	0.021847042	0.024719969
H18	0.01869668	0.027838	0.017200577	0.006952491	H43	0.015630354	0.01224	0.016248196	0.034762457
H19	0.017862007	0.013264	0.023174603	0.003476246	H44	0.020576891	0.01224	0.020606061	0.024719969
H20	0.026226311	0.021803	0.024271284	0.010814986	H45	0.021016193	0.020209	0.017979798	0.011201236
H21	0.01555128	0.023341	0.022914863	0.005793743	H46	0.015015332	0.024251	0.018961039	0.02394747
H22	0.016834041	0.025959	0.017344877	0.011973735	H47	0.027632076	0.022942	0.018759019	0.035921205
H23	0.01504169	0.027496	0.021500722	0.032831209	H48	0.016394739	0.012296	0.018297258	0.007338741
H24	0.015858791	0.015257	0.021385281	0.005021244	H49	0.026621682	0.020608	0.015901876	0.009656238
H25	0.022703111	0.02613	0.022597403	0.020471224	H50	0.015770931	0.016395	0.017950938	0.012359985

Figure 8. Matrix Pij

The calculation for ej, which is the total information entropy value of factor j, is separated into three sections. The part in the formula (6):

$$p_{ij} \cdot \ln p_{ij} \tag{7}$$

is first calculated by multiplying the value of each element in the new matrix by the natural logarithm of itself, which formed another matrix (**Figure 9**).

	C1	C2	C3	C4		C1	C2	C3	C4
H1	-0.069863392	-0.062175517	-0.085481247	-0.105515958	H26	-0.098481773	-0.05619782	-0.070939968	-0.064428699
H2	-0.09043124	-0.094178374	-0.081981027	-0.102593826	H27	-0.07266496	-0.086759164	-0.071990383	-0.041962632
H3	-0.095071644	-0.068632006	-0.085964129	-0.055605694	H28	-0.070481213	-0.084855049	-0.067657548	-0.110287937
H4	-0.07266496	-0.077522789	-0.080991491	-0.117683754	H29	-0.095489791	-0.068280174	-0.084105821	-0.124807573
H5	-0.067395407	-0.081959456	-0.075529488	-0.056898671	H30	-0.073008503	-0.099991045	-0.088517447	-0.101609571
H6	-0.083869307	-0.100576998	-0.069440146	-0.122166537	H31	-0.088825401	-0.085809681	-0.087803036	-0.102593826
H7	-0.068457782	-0.076017762	-0.077313475	-0.106480012	H32	-0.077102196	-0.064176853	-0.072600022	-0.069251774
H8	-0.07133689	-0.05154756	-0.072773789	-0.107439176	H33	-0.064499138	-0.075006854	-0.086365523	-0.036066888
H9	-0.096968852	-0.077356227	-0.080660379	-0.081824318	H34	-0.081626348	-0.076520936	-0.08337325	-0.050312941
H10	-0.063885437	-0.073308232	-0.076466321	-0.081824318	H35	-0.066545854	-0.052724566	-0.081981027	-0.056898671
H11	-0.077359579	-0.101015255	-0.085400637	-0.10357293	H36	-0.094068843	-0.098073805	-0.080494583	-0.123931182
H12	-0.096115225	-0.090661566	-0.075529488	-0.050312941	H37	-0.089162144	-0.053891764	-0.078492396	-0.023202954
H13	-0.088463899	-0.059216677	-0.088992134	-0.100620097	H38	-0.090072973	-0.051941003	-0.077397957	-0.050312941
H14	-0.069351228	-0.07868416	-0.076720915	-0.099625331	H39	-0.076973357	-0.074499092	-0.074157366	-0.05430101
H15	-0.079633223	-0.081472124	-0.079496425	-0.056898671	H40	-0.080317024	-0.093571639	-0.066848824	-0.121278199
H16	-0.062960291	-0.086916916	-0.070500285	-0.09250737	H41	-0.070695555	-0.074499092	-0.07824046	-0.034544532
H17	-0.077230937	-0.088017262	-0.072947372	-0.052984254	H42	-0.063297351	-0.086285061	-0.08353631	-0.091467443
H18	-0.074401742	-0.099697374	-0.069882717	-0.034544532	H43	-0.06499946	-0.053891764	-0.066938886	-0.116774646
H19	-0.071895994	-0.057337181	-0.087245389	-0.019681816	H44	-0.079912139	-0.053891764	-0.079996231	-0.091467443
H20	-0.095489791	-0.083413273	-0.090251832	-0.048957524	H45	-0.081174247	-0.078849423	-0.072251935	-0.050312941
H21	-0.0647495	-0.087703573	-0.086525824	-0.029843434	H46	-0.063044625	-0.090197755	-0.075187516	-0.089369386
H22	-0.068756151	-0.094783094	-0.070324078	-0.052984254	H47	-0.099165387	-0.086601271	-0.074587371	-0.119489284
H23	-0.063128913	-0.098813558	-0.082555649	-0.112163745	H48	-0.067395407	-0.054085363	-0.073207405	-0.036066888
H24	-0.065719326	-0.063814903	-0.082227527	-0.026582854	H49	-0.096530999	-0.0800018	-0.065854728	-0.044806404
H25	-0.085937027	-0.095235322	-0.085642355	-0.079607168	H50	-0.065442846	-0.067397181	-0.072164797	-0.05430101

Figure 9. Pij×ln(Pij) Matrix

The following step is to calculate the sum of each column and the value of the constant k, determined by the equation: [7]

$$k = 1/ln(n) > 0 \tag{8}$$

in which n represents the number of alternatives, or hospitals in this case.

Thus, the value of k is equal to

$$k=1/\ln (50) =0.255622.... \tag{9}$$

and the values of e_j were calculated as shown in **Figure 10.**

	C1	C2	C3	C4	SUM
SUM	-3.892115273	-3.878057076	-7.77017235	-3.758765958	
K	0.255622219				
EJ	-0.994911141	-0.991317554	-1.986228695	-0.960824093	
Dj	1.994911141	1.991317554	2.986228695	1.960824093	8.933281
wj	0.223312245	0.222909975	0.334281272	0.219496508	
	22.33%	22.29%	33.43%	21.95%	

Figure 10. Data table Calculation

The difference between 1 and ej, which is dj, [30]

$$d_j = 1 - e_j \tag{10}$$

decides the information utility value of an index. Its value directly influences the weights. The importance of the factor to the evaluation increases as the information utility value increases, and so does the weight. [26] The values of the weights (wj) were calculated by dividing each individual d_j by the sum of the d_j values, [30]

$$\omega_j = (1 - e_j) / \sum_{j=1}^{n} (1 - e_j) \tag{11}$$

which is 8.933281. The weights of C1, C2, C3 and C4 are 22.33%, 22.29%, 33.43% and 21.95% respectively. As a result, C1, C2, and C4 weigh approximately the same and are of almost equal importance, however, C3, the supply expenditures, is the factor that has the most impact on the quality of a hospital among these four factors. In terms of their weighted values, scores of the 50 hospitals are revealed in **Figure 11**, listed from the highest to the lowest. The top 5 are highlighted.

Hospital Name	Scores
H11	113.7790606
H6	112.5224913
H30	112.3447103
H18	111.0305132
H23	110.3381923
H36	109.1689229
H25	105.045581
H22	103.7200158
H2	103.3831775
H40	101.9470175

……..

H44	50.45140497
H48	50.30798368
H37	50.27497971
H43	50.00355625
H35	49.12827376
H38	48.04203919
H8	47.52749628

Figure 11. Scores of the 50 Hospitals

4.2.2 AHP Method

The Analytic Hierarchy Process (AHP) considers other factors together with mortality to pick the best one out of the Top 5. The goal is decomposed into a hierarchy of criteria and alternatives (see **Figure 12**). The structure of the hierarchy includes the goal, which is the best hospital, the criteria, which are the five factors, and the alternatives, which are the hospitals. [31]

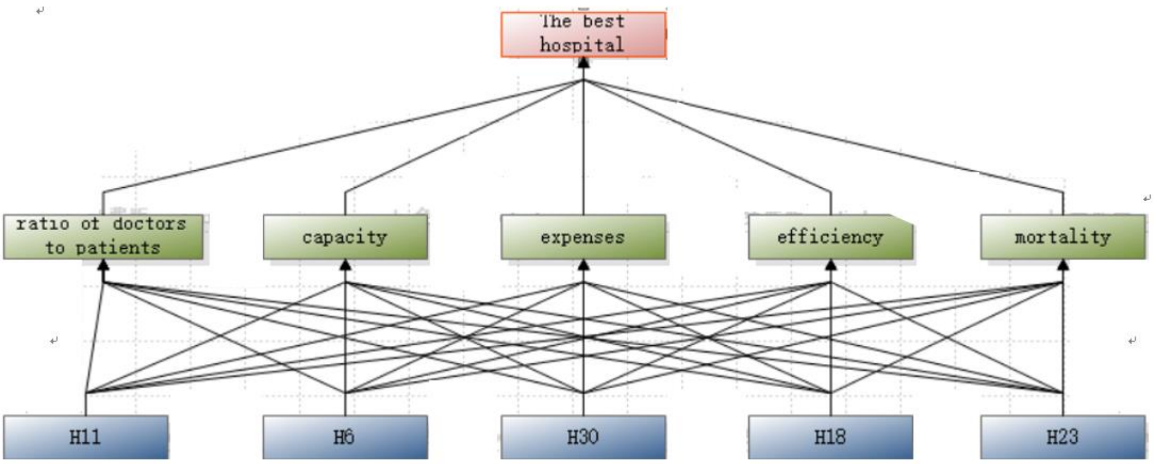

Figure 12. Structure of Hierarchy

4.2.2.1 Single Hierarchical Arrangement

Scale	Degree of preference
1	Equal importance
3	Moderate importance of one factor over another
5	Strong or essential importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Values for inverse comparison

Figure 13. Scale for Comparison [18]

According to the scale for comparison (**Figure 13**), six pairwise comparisons between two criterions are made as shown in **Figure 14.** Instead of subjectively deciding the importance of a factor over another, the data and the weights from 3.2.2 Calculation for ~~m~~ and 4.2.1 Entropy Method are used to objectively determine the importance of the factors. The degree of preference between the two criteria/alternatives with a maximum difference is set to be 9, and the rest is filled in with the following formula and rounded to the nearest whole number:

$$9\times\text{difference}/\text{max difference} \quad (12)$$

The lower triangular matrix is always the reciprocal values of the upper diagonal. [15]

	Criteria				
Matrix A	C1	C2	C3	C4	M
C1	1	1	1/2	1	1/9
C2	1	1	1/2	1	1/9
C3	2	2	1	2	1/7
C4	1	1	1/2	1	1/9
M	9	9	7	9	1
sum	14	14	9 1/2	14	1 1/2

	ratio of doctors to patients				
Matrix B1	H11	H6	H30	H18	H23
H11	1	1/3	2	1	6
H6	3	1	5	4	9
H30	1/2	1/5	1	1/6	4
H18	1	1/4	6	1	5
H23	1/6	1/9	1/4	1/5	1
sum	5.66667	1 8/9	14.25	6.36667	25

	Capacity				
Matrix B2	H11	H6	H30	H18	H23
H11	1	2	4	5	9
H6	1/2	1	2	4	7
H30	1/4	1/2	1	1	5
H18	1/5	1/4	1	1	4
H23	1/9	1/7	1/5	1/4	1
sum	2.06111	3.89286	8.2	11.25	26

	Expense				
Matrix B3	H11	H6	H30	H18	H23
H11	1	7	1/2	7	1
H6	1/7	1	1/9	1	1/6
H30	2	9	1	9	3
H18	1/7	1	1/9	1	1/6
H23	1	6	1/3	6	1
sum	4.28571	24	2	24	5.33333

	Efficiency				
Matrix B4	H11	H6	H30	H18	H23
H11	1	1/2	1	7	1
H6	2	1	3	9	1
H30	1	1/3	1	6	1
H18	1/7	1/9	1/6	1	1/8
H23	1	1	1	8	1
sum	5.14286	3	6.16667	31	4.125

	Motarlity				
Matrix B5	H11	H6	H30	H18	H23
H11	1	1/5	1	4	1/4
H6	5	1	6	9	1
H30	1	1/6	1	3	1/6
H18	1/4	1/9	1/3	1	1/8
H23	4	1	6	8	1
sum	11.25	2 1/2	14.3333	25	2 1/2

Figure 14. Pairwise Comparisons Between Each Two Criterion

The normalized scores in the new matrices (**Figure 15**) are the quotients of each element and their column sums. Then the weighted matrix (the W column) was generated by normalizing the row vectors by dividing the row sum by the total sum of the sum column. [15]

New Matrices: Normalized

	Criteria							
Matrix A	C1	C2	C3	C4	M	sum	W	consistency
C1	0.0714	0.0714	0.0526	0.0714	0.0753	0.3422	0.0684	5.0854
C2	0.0714	0.0714	0.0526	0.0714	0.0753	0.3422	0.0684	5.0854
C3	0.1429	0.1429	0.1053	0.1429	0.0968	0.6844	0.1369	4.6978
C4	0.0714	0.0714	0.0526	0.0714	0.0753	0.3422	0.0684	5.0854
M	0.6429	0.6429	0.7368	0.6429	0.6774	3.3428	0.6686	5.1969
							CI=	0.007553
							RI=	1.12
							CR=	0.006744

	ratio of doctors to patients C1							
Matrix B1	H11	H6	H30	H18	H23	sum	W	consistency
H11	0.1765	0.1760	0.1404	0.1571	0.2400	0.8898	0.1780	5.3241
H6	0.5294	0.5279	0.3509	0.6283	0.3600	2.3964	0.4793	5.5316
H30	0.0882	0.1056	0.0702	0.0262	0.1600	0.4502	0.0900	5.0283
H18	0.1765	0.1320	0.4211	0.1571	0.2000	1.0866	0.2173	5.6707
H23	0.0294	0.0587	0.0175	0.0314	0.0400	0.1770	0.0354	5.2053
							CI=	0.088007
							RI=	1.12
							CR=	0.078578

	Capacity C2							
Matrix B2	H11	H6	H30	H18	H23	sum	W	consistency
H11	0.4852	0.5138	0.4878	0.4444	0.3462	2.2773	0.4555	5.1973
H6	0.2426	0.2569	0.2439	0.3556	0.2692	1.3682	0.2736	5 2/9
H30	0.1213	0.1284	0.1220	0.0889	0.1923	0.6529	0.1306	5
H18	0.0970	0.0642	0.1220	0.0889	0.1538	0.5259	0.1052	5
H23	0.0539	0.0367	0.0244	0.0222	0.0385	0.1757	0.0351	5
							CI=	0.031483
							RI=	1.12
							CR=	0.028110

	Expense C3							
Matrix B3	H11	H6	H30	H18	H23	sum	W	consistency
H11	0.2333	0.2917	0.2432	0.2917	0.1875	1.2474	0.2495	5.0431
H6	0.0333	0.0417	0.0541	0.0417	0.0313	0.2020	0.0404	5
H30	0.4667	0.3750	0.4865	0.3750	0.5625	2.2657	0.4531	5.1398
H18	0.0333	0.0417	0.0541	0.0417	0.0313	0.2020	0.0404	5
H23	0.2333	0.2500	0.1622	0.2500	0.1875	1.0830	0.2166	5.0871
							CI=	0.015739
							RI=	1.12
							CR=	0.014053

	Efficiency C4							
Matrix B4	H11	H6	H30	H18	H23	sum	W	consistency
H11	0.1944	0.1698	0.1622	0.2258	0.2424	0.9946	0.1989	5.0839
H6	0.3889	0.3396	0.4865	0.2903	0.2424	1.7477	0.3495	5.1764
H30	0.1944	0.1132	0.1622	0.1935	0.2424	0.9058	0.1812	5.0899
H18	0.0278	0.0377	0.0270	0.0323	0.0303	0.1551	0.0310	5 1/9
H23	0.1944	0.3396	0.1622	0.2581	0.2424	1.1967	0.2393	5.0853
							CI=	0.027076
							RI=	1.12
							CR=	0.024175

	Motarlity M							
Matrix B5	H11	H6	H30	H18	H23	sum	W	consistency
H11	0.0889	0.0807	0.0698	0.1600	0.0984	0.4977	0.0995	5.0308916
H6	0.4444	0.4036	0.4186	0.3600	0.3934	2.0201	0.4040	5.1901048
H30	0.0889	0.0673	0.0698	0.1200	0.0656	0.4115	0.0823	5.1024029
H18	0.0222	0.0448	0.0233	0.0400	0.0492	0.1795	0.0359	5.0247624
H23	0.3556	0.4036	0.4186	0.3200	0.3934	1.8912	0.3782	5.1857218
							CI=	0.0266942
							RI=	1.12
							CR=	0.0238341

Figure 15. Normalized Scores and Consistency Analysis

Consistency analysis

To evaluate and check the consistency of judgement, λmax first needs to be calculated. Therefore, the consistency column was computed with the matrix multiplication function (MMULT) to multiply two matrices: each row of the pairwise comparison and the weighted matrix (**Figure 16**), and the product is divided with its corresponding weight like shown in (**13**). The average of the five consistency vectors will then be λmax (**14**). [33] [25]

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \times \begin{bmatrix} W_{11} \\ W_{12} \\ W_{13} \end{bmatrix} = \begin{bmatrix} C_{V11} \\ C_{V21} \\ C_{V31} \end{bmatrix}$$

Figure 16. Matrix Multiplication [15]8

$$\frac{(\overrightarrow{\mathbf{A}}\overrightarrow{\mathbf{w}})_i}{w_i} \quad (13)$$

$$\lambda_{\max} = \frac{1}{n}\sum_{i=1}^{n}\frac{(\overrightarrow{\mathbf{A}}\overrightarrow{\mathbf{w}})_i}{w_i} \quad (14)$$

CI (the consistency index) which measures the deviation was calculated using the equation, [25]

$$CI = \frac{\lambda_{\max} - n}{n-1} \quad (15)$$

For n=1, 2...,10, the values of RI (the random index) are given by Thomas L. Saaty [32], shown in **Figure 17**, and has a value of 1.12 when n equals five, which is true in this case.

n	2	3	4	5	6	7	8	9	10
RI	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

Figure 17. Values of RI [18]

The consistency ratio (CR) can be calculated by dividing CI with RI (**16**), and a CR of lower than 0.1 is considered acceptable. [16] A lower CR means a higher consistency. All six matrices all pass the consistency examination. [16]

$$CR = \frac{CI}{RI} \tag{16}$$

4.2.2.2 Total Taxis of Hierarchy

Criteria / Alternatives	C1	C2	C3	C4	M	Final weight
	0.0684	0.0684	0.1369	0.0684	0.6686	
H11	0.1780	0.4555	0.2495	0.1989	0.0995	0.15767
H6	0.4793	0.2736	0.0404	0.3495	0.4040	0.35109
H30	0.0900	0.1306	0.4531	0.1812	0.0823	0.14454
H18	0.2173	0.1052	0.0404	0.0310	0.0359	0.05372
H23	0.0354	0.0351	0.2166	0.2393	0.3782	0.30373

CI=0.03003	RI=1.12	CR=0.02681

Figure 18. Weights of Criteria and Alternatives [34]

Since the single hierarchies were all examined to be consistent, the analysis of the overall weights of the five alternatives can be continued. The weights of the criteria and alternatives determined in 4.2.2.1 were shown in **Figure 18**. The final weights are the sum of the products of bj and cij, [34] in which the b and c represent the criteria and alternative levels, so in our case are the factors and the hospitals. The calculations of the overall CI and RI are: [25]

$$RI = \sum_{j=1}^{5} b_j RI_j \tag{17}$$

$$CI = \sum_{j=1}^{5} b_j CI_j \tag{18}$$

Through calculation, the overall CR ratio, which is 0.0268, is also lower than 0.1, which means the general consistency is acceptable and the weights are valid. Thus, Hospital 6 has the best quality since it has the highest weight, which is 35.11%.

5. Memo

There are myriad of factors in choosing a good hospital. However, if we were to consider them all when picking a hospital, it would complicate the progress much more than is needed. So, our team decided on five of the most important factors: mortality, ratio, capacity, expenditures, and efficiency. Out of those five, mortality and expenditures are found to be more important than the rest.

With regards to mortality rates, while on the surface may be the lower the mortality rates the better, that is not the case at all. When looking at mortality rates, we must look at the evitable and inevitable death ratios as well. Even if a hospital has high mortality rates, it might not have been due to the quality of the hospital as a portion of the deaths could have been inevitable. Likewise, a hospital with low mortality might not be a good hospital if a good number of the deaths could have been avoided. The mortality rates also need to be looked at with regards to the total number of patients in a hospital, as a percentage, since just the numbers alone is not a good representation. There are many factors that influence whether a death is considered evitable or inevitable, and we picked some of the most influential ones: age, diseases, accidents, and resource shortages. For age, if two hospitals have the same mortality rate and percentage, then we can look at the population of the area the hospital is in. If the population is mostly made up of seniors, there's a good chance that the hospital in that area will have a higher number of inevitable deaths than evitable deaths, as an elderly person is more likely to have a higher chance of death than a younger person when going through the same procedures. Diseases also have a huge impact on inevitable death rates. As the leading cause of death all around the world, diseases take away more lives than anything else; and more often than not, doctors find themselves unable to do anything to salvage the situation. Those who pass away under these circumstances are counted towards inevitable deaths. The same can be said with accidents, as another leading cause of death. For resource shortages, we are aiming more towards organs and the like rather than medicines and equipment. There are hundreds of thousands of people waiting for suitable organ donors each year, while there are only tens of thousands of donors. Statistics have shown that 20[28] people would die each day in Canada just from waiting for organ transplants alone. Thus, those who die from resource shortages also make up a large amount of inevitable deaths.

When comparing hospitals, we should also look at the doctor-to-patient ratio. When comparing the amount of time a doctor with a higher number of patients spends on each patient with a doctor who has a lower number of patient, the result is obvious – the doctor with the lower number of patients spends more time on average on each patient. While a doctor with a large number of patients may be just as capable – if not even more – than the one with fewer patients, you cannot deny the fact that sessions with each patient will be shorter and more rushed. No matter how competent a person is, if there is not enough time to showcase their ability, what's the use? When more time is spent on the patient, the more information regarding the illness/condition will be revealed, and thus the treatment may also speed up. The patients will not only be treated quicker, they will also be left with a good impression of the doctor, and in turn, the hospital.

Capacity is also a very important factor when looking for a good hospital.

When a hospital has more room, they have the ability to take in more people. Not only can the hospital treat more people, they can also hire more staff, house more equipment, and so on. Usually – but not always – a bigger hospital means better treatment, as they would have the money to purchase not just more, but better equipment. Not just the equipment will be better, the staff a bigger hospital would hire would need to have a better resume and more experience as well.

As for the equipment, we will need to look into how much a hospital spends on maintaining, cleaning, buying, and replacing their equipment. Having better working, cleaner, more, and newer equipment such as MIR, CAT scan, ultrasound, ECG, and so on will mean the patient will not only have access to all these devices, but they will be of a better quality as well. Not just the equipment for treatment, but the equipment for cleaning is equally as important. The equipment and instruments that have come in contact with bodily fluids would need to go through several rounds of cleaning and sterilizing before they are fit to be used again to prevent infections and transmissions of diseases. Having better quality cleaning equipment would lessen the chance of incidents of infection and transmission through particular instruments.

On top of doctor-to-patient ratio, capacity, and equipment expenses, there is also the efficiency of the doctors. There is no doubt that good doctors represent good hospitals. A more efficient doctor would be able to treat more patients – when considering that the patient's medical conditions and illnesses are similar – in a same amount of time when compared to a less efficient doctor. As the time taken for each treatment lessens, more people can be treated, and the waiting time will also go down significantly.

While these five factors are all very important, we cannot deny that mortality and expenditures are more important than the others. After calculations, we have found specific percentages of how much the five factors weigh. Without considering mortality, diseases weigh 22.33%, accidents weigh 22.29%, expenditures weigh 33.43%, and efficiency weighs 21.95%. However, when you add in mortality, diseases, accidents and efficiency all weigh 6.84%, expenditures weigh 13.69%, and mortality weighs 66.86%. From this, you can see that mortality has the utmost importance in deciding a hospital’s quality, expenditure is next, while the others are all in third place.

In the end, to choose a good hospital, you must consider several aspects before deciding: mortality rates, doctor-to-patient ratio, capacity, equipment, and efficiency. Each of those factors represent the quality of a hospital, and when choosing, while it would be ideal to have a hospital that excels in all aspects, it would be extremely difficult for a hospital to achieve perfection, so as long as the hospital of your choice is not terrible at any of the above aspects, it would be fine.

Best regards,

Team # 2018055

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