Team Control Number

2022038

2022

The International Mathematical Modeling Challenge (IM^{2}C)
Summary Sheet

As air transportation has become more and more popular over the years, we want to find ways to minimize the total boarding and disembarking times on aircraft. Time is money, so it is important for airline companies to save as much time as possible. However, there are many boarding and disembarking methods to choose from, so we build mathematical models to determine which ones are the most time-effective and practical.

Our Narrow Body Boarding Time Model (BTM) Based on Gantt Chart uses several sub-models and the Gantt chart to find the total boarding time through a bottom-up approach. After computing our sub-models (Aisle Walking Model Based on Differential Equations, Luggage Interruptions Model, and the Seating Model - Including Seating Interruptions), we calculate individual passenger's total boarding time in the Single-Passenger Boarding Time Model. Then, using a Gantt Chart, we find the total boarding time for the narrow body aircraft. Through the same ideology, we build our Narrow Body Disembarking Time Model (DTM) Based on Gantt Chart in order to find the total disembarking time.

After we build our BTM and DTM, we use Monte Carlo Simulations to model the boarding and disembarking processes with the presence of random variables and calculate the boarding and disembarking times with our models. In the simulations, we generate a sequence of passengers and carry out every "step" of passengers' actions with our BTM and DTM.

We apply our BTM and DTM to the narrow body aircraft with the different boarding and disembarking methods. We perform a sensitivity analysis which showed that out of the three boarding methods in the problem, the **Boarding by Seat** boarding method is the best, for it has the lowest average boarding times with varying parameters.

In addition to the three boarding methods given in the problem, we identify two other boarding methods: the Reverse Pyramid and the Steffen Method. We find that the optimal idealistic boarding method is the **Steffen Method**. However, it is very difficult to implement the Steffen Method in a real-life setting, so we recommend the optimal practical boarding method: the **Reverse Pyramid** as it is viable in real-life practice and has the least boarding time. We also found the optimal practical disembarking method is the **Steffen Method**, since there's no queue for disembarking.

We extended our BTM and DTM to two other aircrafts: we built Flying-Wing Models (FWM) and Two-entrance, Two-aisle Models (TTM) Based on Queuing Networks. We consider boarding and disembarking as a queuing network and modify our BTM and DTM to create FWM and TTM. We found that the optimal practical boarding and disembarking methods for those two aircrafts are **both Boarding by Seat**.

We also considered the boarding and disembarking processes of the three aircrafts under pandemic situations, where only 30%, 50%, or 70% of the seats are open. The optimal practical boarding methods for the three aircrafts are **Reverse Pyramid**, **Random**, and **Random** for the narrow body, Flying Wing, and Two-entrance, Two-aisle aircraft.

With our models and results, we have found optimal boarding and disembarking methods that will remove the headaches of airline executives when trying to minimize boarding and disembarking times. We envision a future with less time wasted during boarding and disembarking processes, and this is our contribution to it.

Keywords: Aircraft Boarding and Disembarking Time, Bottom-up Approach, the Gantt Chart, Differential Equations, Monte Carlo Simulation, Queuing Networks

FROM THE DESK OF

Team 2022038

April 18, 2022

Dear Airline Executive,

This is Team 2022038 writing to inform you about our most recent research. Through simulation ran with computer models designed by us, we have determined the best methods to board and disembark for multiple types of aircrafts.

The most important type of aircraft in our research is the standard narrow-body aircraft used by airlines all around the world. With our mathematical models, we concluded that Reverse Pyramid is the best way to board this type of aircraft for your airline. Reverse Pyramid is the perfect balance between efficiency and simplicity: it blends advantages of Boarding by Section (minimize time dealing with luggage) and Boarding by Seat (minimize time dealing with seating problems) while remaining practical. That said, the absolute fastest way to board is actually the Steffen Method. However, this method is too idealized and will take too much time for passengers to form the correct line before boarding.

We also varied the number of passengers not following the prescribed boarding method and the number of luggage passengers carry, and we discovered that the Boarding by Seat method actually had the lowest boarding time when those parameters are changed.

On the side of disembarking, we would recommend the Steffen Method. As stated above, it is the quickest method to organize the passengers to leave an aircraft, and since it is practical for disembarking (there is not a need to make a queue), we highly recommend this method.

Two other aircrafts that we applied our models to are the Flying-Wing and Two-Entrance, Two-Aisle Aircraft. We discovered that the best boarding method for them both is the Boarding by Seat method.

In light of the current pandemic, we also applied our models to the three aircrafts with limited number of passengers to 30%, 50%, and 70%. For the three aircrafts narrow body, Flying Wing, and Two-entrance, Two-aisle, the following boarding methods are the best, respectively: Reverse Pyramid, Random, and Random.

We would like to thank you for taking your time to read our letter, and we sincerely hope that our findings and recommendations can improve the airline industry in the slightest way possible.

Sincerely yours,

Team 2022038

Contents

1 Introduction

1.1 Background

In 2019, around 4.5 billion passengers worldwide took 42 million airplane flights: airplanes have become a common means of transportation. [1] With many passengers on each flight, airline companies have to carefully plan out their flights so that they maximize their time because time is money. Therefore, it is demanding for airlines to find optimal strategies to minimize the time spent on the two most time-consuming operations, the boarding and disembarking processes.

Different airlines use a variety of boarding and disembarking methods to minimize the total time taken for these operations. There are unstructured methods such as the random method, and there are structured methods such as boarding by section and boarding by seat. [2] Moreover, some passengers may not follow the method implemented by the airline. Since there are so many different ways of boarding and disembarking, it is crucial to pick out the **best** method that is the most efficient and practical.

In order to evaluate the different boarding and disembarking methods, the movement of passengers must be considered. It takes time for passengers to walk to their seat and get seated. In addition, passengers may encounter interruptions (e.g., putting their luggage in the overhead-bins or taking the luggage down) that cause them to stop in the aisle and forbade others behind them to move forward. These motions contribute to the total boarding and disembarking time, so it is important to consider these factors when investigating which boarding and disembarking method saves the most time.

1.2 Problem Restatement

The overarching goal is to find a boarding and disembarking strategy that will be both time efficient and practical on aircraft.

1. We should build a model to calculate the total time of boarding and disembarking. In this model, we need to consider different factors including the interruptions passenger face and passengers who are not following the prescribed boarding or disembarking method.

2. We want to apply our model to a standard ”narrow-body” aircraft and determine the optimal boarding and disembarking methods. When applying, we need to consider some practical issues including, how does different broadly used boarding method influence out model, how does the percentage of passengers disobey the instructions and the number of carry-on bags impact the different methods, and how does more luggage influence the model.

3. We are asked to adapt our model from the previous standard ”narrow-body” aircraft to both the ”Flying Wing” aircraft and the ”Two-entrance, Two-aisle” passenger aircraft. We are also asked to recommend optional boarding and disembarking methods for each aircraft.

4. We need to consider how the limitations towards the number of passengers under different pandemic situations affect the optimal boarding and disembarking method on the three aircraft.

5. We are writing a one-page letter to the airline executive to explain our results in a non-mathematical way and provide some suggestions for the boarding and disembarking processes.

1.3 Our Work

For a narrow body passenger aircraft, we develop a boarding model and a disembarking model based on the Gantt chart to find the total boarding and disembarking time respectively. Both models have four sub-models: a single-passenger boarding time model, an aisle walking model based on differential equations, a luggage interruptions model, and a seating model. After we determine values for these sub-models and thus model, we use Monte Carlo Simulations to find the total boarding and disembarking time on a narrow body aircraft and compare those of different boarding and disembarking methods. Furthermore, we performed sensitivity analysis to test the stability of those boarding and disembarking methods.

Then, we apply our models to other aircraft and conditions. We modify our Narrow Body Boarding and Disembarking Time Model to create the boarding and disembarking models of a Flying Wing aircraft and a Two-entrance, Two-aisle aircraft. Moreover, for all three aircraft, we revise our models for pandemic situations where only a percentage of passengers can board. With these models, we obtain results and found their optimal boarding and disembarking strategies.

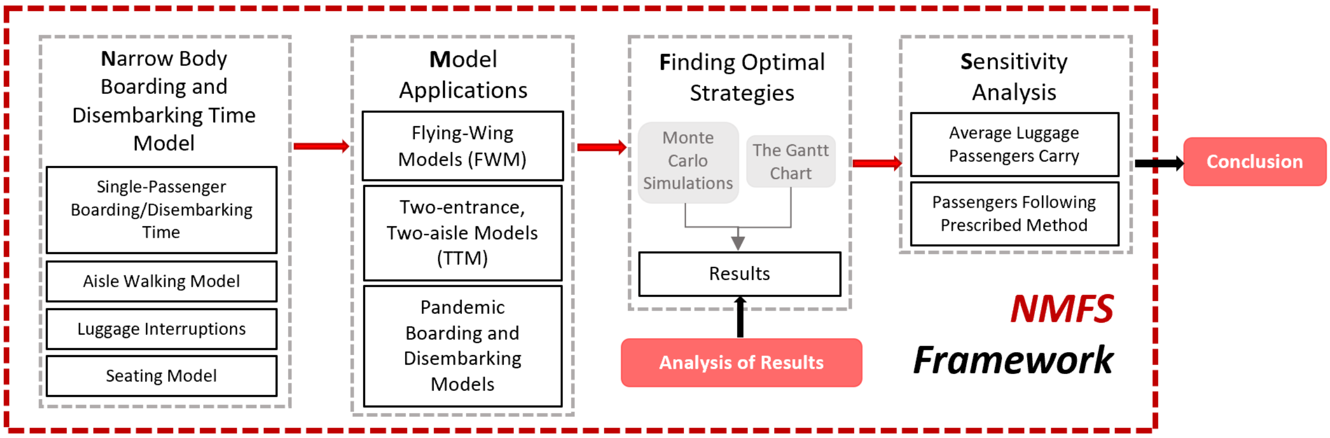


Figure 1.1: Flowchart of Our Work

2 Variables and Meanings

Table 2.1: Variables Table for Boarding Model of Narrow Body Aircraft

Variables	Description
$T^n_{boarding}$	Total boarding time of the n^{th} passenger
T^n_{aisle}	Total time the n^{th} passenger walks in the aisle when boarding
$T^n_{luggage}$	Total time the n^{th} passenger deals with luggage interruptions when boarding
$T^n_{seating}$	Total time it takes for the n^{th} passenger to be fully seated
$Position^n_t$	Position of the n^{th} passenger at time t
$P^n_{static-t}$	Whether the n^{th} passenger is stopped in the aisle at time t
$P^n_{arrival-t}$	Whether the n^{th} passenger has arrived at their desired row at time t
Row^n	Desired row number of the n^{th} passenger
D_{seat}	Distance between each row on an aircraft
P^n_{static}	Whether the n^{th} passenger has stopped
$P^n_{arrival}$	Whether the n^{th} passenger arrived at Row^n while $(n+1)^{th}$ passenger boards
$T^n_{luggage}$	Time the n^{th} passenger spends dealing with luggage interruptions
$R^{x2}_{x_1}$	Recursive replacement of the x^{th}_1 passenger with the x^{th}_2 passenger
T_{put}	Time takes to put one's carry-on bag in the overhead bin
$N^n_{luggage}$	Number of carry-on bags the n^{th} passenger has
$N^{existing}_{luggage}$	Number of existing carry-on bags in the overhead bin of the n^{th} passenger
α_i	Whether the i^{th} passenger's desired row is the n^{th} passenger's row
$N^{max}_{luggage}$	Maximum number of carry-on bags in the n^{th} passenger's overhead bin
$N_{passenger}$	Number of passengers in the N^{th} row
$T_{arrange}$	Time it takes to arrange one carry-on bag in the overhead bins
β	Amount of time it takes to re-arrange an entire half-row of luggage
$T^n_{seating}$	Time the n^{th} passenger spent in order to be fully seated
$T^n_{interruption}$	Time the n^{th} passenger spent dealing with seating interruptions
V_{aisle}	Speed of a passenger to move a grid in the aisle
V_{seat}	Speed of a passenger to move a grid in the seats
N^{seats}_{grids}	Number of grids that the n^{th} passenger has to walk in order to be fully seated
$N^{interruption}_{grids}$	Number of grids that the n^{th} passenger has to walk in order to exit the aisle
$T^n_{disembarking}$	Total disembarking time of the n^{th} passenger
B^{n-1}_n	Rows with at least one passenger between the n^{th} and $(n-1)^{th}$ passenger
AV_i	Whether there is an available passenger in row i
RD^n_{n+x}	Recursive replacement of the n^{th} passenger with the $(n+x)^{th}$ passenger
T_{put}	Time to take down one carry-on bag from the overhead bin
$T^n_{f-boarding}$	Total boarding time of the n^{th} passenger in the Flying Wing aircraft
$T_{f-disembarking}$	Total disembarking time of the n^{th} passenger in the Flying Wing aircraft
T^n_{top}	Time the n^{th} passenger spent walking in the top aisle when boarding
T^n_{d-top}	Time the n^{th} passenger spent walking in the top aisle when disembarking
A^n_f	Number of six-grids that the n^{th} passenger is away from the entrance
$RF^{x_1}_{x_2}$	Replace the x^{th}_1 passenger with the x^{th}_2 passenger in the top aisle
P^n_{new}	Whether there is a new passenger coming in to top aisle as the n^{th} passenger
RDF^{n+1}_n	Recurrence to make the n^{th} passenger the $(n+1)^{th}$ passenger
$T^n_{t-boarding}$	Total boarding time of the n^{th} passenger in the Two-entrance aircraft
$T_{t-disembarking}$	Total disembarking time of the n^{th} passenger in the Two-entrance aircraft
$T^n_{entrance-aisle}$	Time the n^{th} passenger spent in the entrance aisle when boarding
$T^n_{d-entrance-aisle}$	Time the n^{th} passenger spent in the entrance aisle when disembarking
A^n_t	Number of grids between the n^{th} passenger's aisle and the entrance

2.1 Passenger Interruptions

When boarding and disembarking, passengers face different types of interruptions that will cause them to stop in the aisle. We note the two main interruptions that are very time-consuming for passengers: seat interruptions and luggage interruptions. [3]

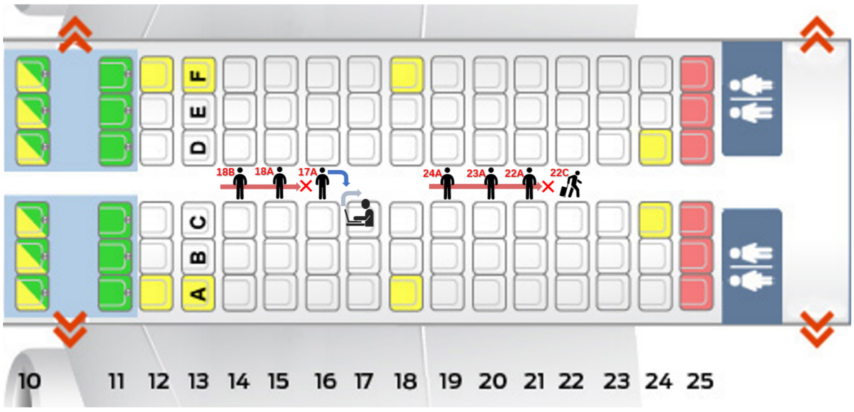


Figure 2.1: Interruptions that Cause Delay in the Queue

Luggage interruptions take place when a passenger has arrived at their row destination with luggage. They have to put their luggage into the overhead bins in their respective half-row, causing them to stop in the aisle. Therefore, they would cause a delay for the passengers behind them as seen in row 22 of Figure 2.1.

Seat interruptions take place when a passenger has arrived at their row destination and their seat is a window or middle seat, but someone else is sitting in the middle or aisle seat such that they are blocking the arrived passenger from sitting down. For example in row 17 of Figure 2.1, when someone that just arrived has a window seat, but someone else is already sitting in the aisle seat in the same row, that person who is sitting in the aisle seat must move out of their seat for the window seat person to move in. Then, the aisle seat person can return to their seat. Therefore, this will cause a seat interruption. Other seat interruptions like this may form as well.

2.2 Boarding Strategies

In addition to the three boarding methods given in the problem, we identified two other boarding strategies: Reverse Pyramid and the Steffen Method.

Our five boarding strategies:

- **Random**: Unstructured boarding. Not depicted in Figure 2.2 because the random boarding method is different every time.
- **Boarding by Section (Back to Front)**: Boarding from the aft section (row 23-33), then the middle section (12-22), and finally the bow section (row 1-11).
- **Boarding by Seat**: Boarding window seats (A and F) first, then middle seats (B and E), and finally aisle seats (C and D).
- **Reverse Pyramid**: Boarding passengers from the outer back to the inner front of cabin, like a combination of Boarding by Section and Boarding by Seat method. [4] [5]

- **Steffen Method**: Suggested by astrophysicist Jason Steffen, this is a special boarding method where people board in the following order from back to front: right odd numbered window seat, left odd numbered window seat, right even numbered window seat, left even numbered window seat, right odd numbered middle seat, and so on. [6]

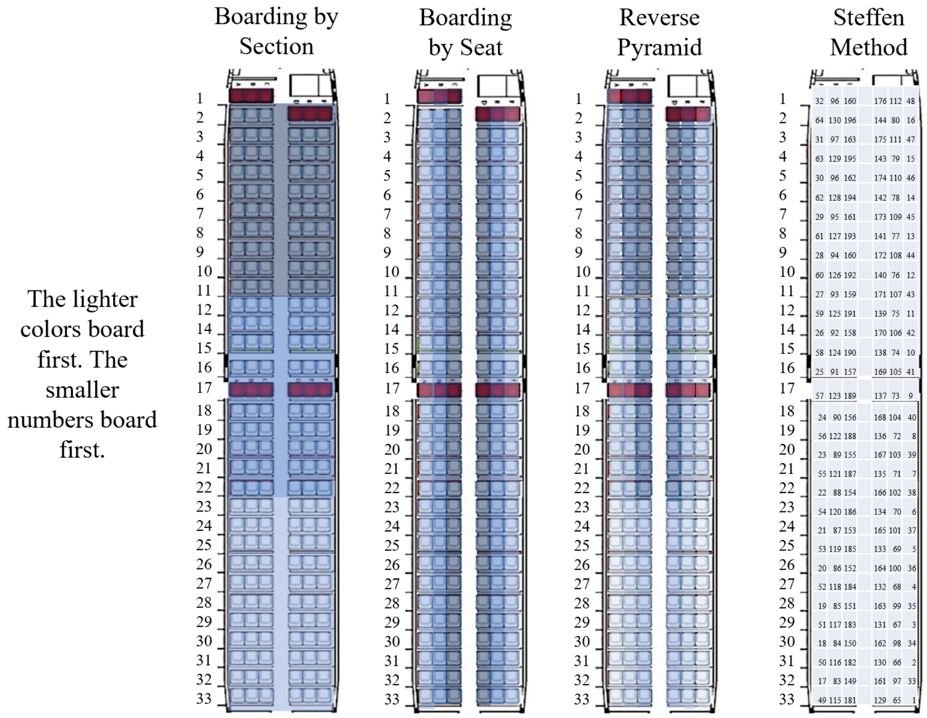


Figure 2.2: Different Boarding Methods Except Random

3 Model Assumptions

- **Assumption 1**: For all boarding methods, we will be able to make a queue before the boarding process with all the passengers, which means that we will not account for late passengers in our boarding process.

  Justification: Late passengers will most likely miss the general boarding process and will board after everyone has done so, so they will not be an influence in the general boarding time.

- **Assumption 2**: We will only consider the passengers of the Economy Class and will not consider the passengers of other classes as a part of the boarding time.

  Justification: Passengers of non-economic class will have boarded before the Economy Class passengers, so they will not be accounted for as a part of the general boarding group.

- **Assumption 3**: Every passenger's speed will be constant and calculated based on the average speed of passengers. We will not consider the effects of age and physical fitness on a passenger's speed, and we will not consider the acceleration of the passengers, for their velocity will remain zero.

  Justification: Moving in an airplane does not require a lot of physical strength for passengers, so we assume that all passengers would be able to undertake the average speed. Moreover, it would not be necessary for passengers to accelerate in an aircraft due to the minimal space.

- **Assumption 4**: In the “Flying Wing” Aircraft and the ”Two-entrance, Two-aisle” aircraft, two flight attendants at the entrances will guide passengers to the aisle they will find their seats in.

  Justification: Usually, there are flight attendants at the entrance of an aircraft to provide guidance to passengers. They will not cause a significant time delay because the speed of passengers is relatively slow, so they will have time to guide all passengers.

- **Assumption 5**: In pandemic situations, passengers will have to be socially distanced in the queue and in seats of the aircraft. In each row, there will be proportionally less passengers.

  Justification: In order to prevent the spread of the pandemic in the aircraft, passengers must sit further away from each other in the rows and distance from each other in the aisle.

- **Assumption 6**: Two passengers will not be able to be in the same row in the aisle, and passengers cannot squeeze past each other in the aisle. Similarly, passengers cannot be walking in the same seat at the same time.

  Justification: The aisle is designed for one person only, and it is narrow, so it is highly unlikely that two passengers will be able to fit in the same row of the aisle simultaneously. Similarly, the seats are small and would not fit two passengers at once.

- **Assumption 7**: Passengers will always keep note of their surroundings and will not miss their designated row. Therefore, no passenger would have to move backwards in the aisle of the aircraft.

  Justification: It’s highly unlikely that a passenger will miss their desired row, for that is their only goal in the boarding process.

4 Narrow Body Boarding Time Model Based on Gantt Chart

To model the boarding process in a narrow body aircraft, we construct a bottom up model based on the Gantt chart: the Narrow Body Boarding Time Model (BTM). We consider the boarding time of each individual passenger on a micro scale, and using the Gantt chart, we find the total boarding time defined below. [7]

Definition 4.1. *Total Boarding Time: The time between when the first passenger entered the aircraft to when the last passenger is completely seated.*

We generate Gantt charts to show the different passenger activities against time, which would help us find the total boarding time. Overall, the Gantt chart shows a well and logically simulated boarding process.

We use the following flowchart to guide us when calculating the total boarding time:

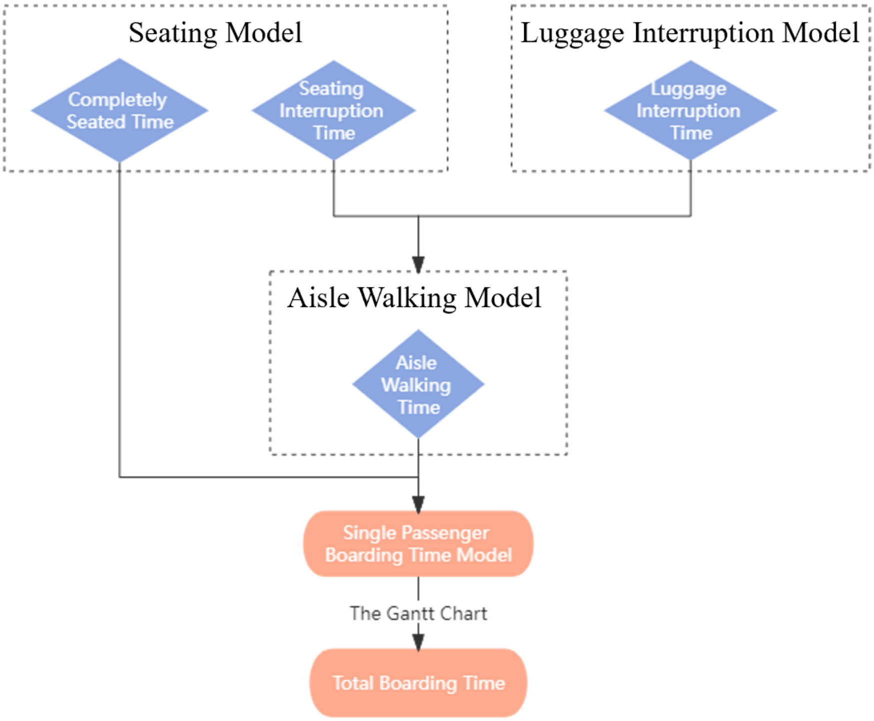


Figure 4.1: Algorithm of the Boarding Model

4.1 Single Passenger Boarding Time Model

Our main model is the single passenger boarding time model, which uses some sub-models explained later in this section.

$$T^n_{boarding} = T^n_{aisle} + T^n_{luggage} + T^n_{seating} \tag{4.1}$$

- $T^n_{boarding}$ is the total boarding time of the n^{th} passenger, from when they entered the aircraft to when they are seated in their respective seat. This considers all the different processes a passenger would encounter in an aircraft when they try to be seated.
- T^n_{aisle} is the total time that the n^{th} passenger spends in the aisle walking to their seat.
- $T^n_{luggage}$ is the total time that the n^{th} passenger spends dealing with their luggage if any.
- $T^n_{seating}$ is the total time that it takes for the n^{th} passenger to be seated after they have reached their desired row.

4.2 Aisle Walking Model Based on Differential Equations

We consider a passenger's speed in the aisle to be directly related to the passenger before them, for each passenger is following the passenger before them. At the entrance, passengers will enter in one-by-one, and there will be no distance between them grid-wise (each row in the aisle creates a grid). The following differential equations model takes into consideration of a passenger's motion in the aisle.

$$\frac{dPosition^n_t}{dt} = P^{n-1}_{static-t} \cdot P^{n-1}_{arrival-t} \cdot P^n_{arrival-t} \cdot V_{aisle} \tag{4.2}$$

$\dfrac{dPosition_t^n}{dt}$ is the speed of the n^{th} passenger at time t, which is either V_{aisle} or 0. Whether the n^{th} passenger is in motion or not solely depends on the $(n-1)^{th}$ passenger and themselves. P_{static}^{n-1}, $P_{arrival}^{n-1}$, and $P_{arrival}^{n}$ are binary variables. If the $(n-1)^{th}$ passenger arrived, $P_{arrival}^{n-1} = 0$. If the $(n-1)^{th}$ passenger stopped (not because they arrived), $P_{static}^{n-1} = 0$. Finally, if the n^{th} passenger arrived at their desired row, they would stop as well. These variables determine if the n^{th} passenger stopped or not.

$$P_{static}^{n-1} = \begin{cases} 1, \text{ if } \dfrac{dPosition_t^n}{dt} = 0 \& Position_t^{n-1} \neq row_t^{n-1} \\ 0, \text{ otherwise} \end{cases} \tag{4.3}$$

$$P_{arrival}^{n} = \begin{cases} 1, \text{ if } Position_t^n = row^n \\ 0, \text{ otherwise} \end{cases} \tag{4.4}$$

If the passenger is in motion, they would walk at the average passenger speed to walk in the aisle: V_{aisle}. We found its value to be $0.36 m/s$ [8].

As the differential equations model above models the n^{th} passenger's motion in the aisle, we derive the following equation to find the total time the n^{th} passenger spends in the aisle.

$$T_{Aisle}^{n} = \frac{Row^n \cdot D_{seat}}{V_{aisle}} + P_{static}^{n-1} \cdot T_{static}^{n-1} + P_{arrival}^{n-1} \cdot (T_{interruption}^{n-1} + T_{luggage}^{n-1}) + R_{n-2}^{n-1} \cdot P_{arrival}^{n-1} \tag{4.5}$$

The above equation shows the total time a passenger takes in aisle. It is the sum of walking time without congestion, the time to stop when the $(n-1)^{th}$ passenger stopped, the time to stop when the $(n-1)^{th}$ passenger arrived and deals with interruptions, and the time to follow the $(n-2)^{th}$ passenger after the $(n-1)^{th}$ passenger left the aisle (note this process is recursive though).

For the first passenger, $T_{Aisle}^{1} = \dfrac{Row^1 \cdot D_{seat}}{V_{aisle}}$ because there is no congestion before the first passenger.

4.2.1 Recursive Replacement

Once the $(n-1)^{th}$ passenger has been seated, we consider the n^{th} passenger with respect to the $(n-2)^{th}$ passenger. Therefore, we write the following recursive replacement to find the n^{th} passenger's boarding time when they have to follow another passenger in the aisle. In the following recursive sequence, we use $n-2$ and $n-1$ as variable values to plug in, but they are just placeholders.

- The superscript of R is the passenger that is leaving the aisle.
- The subscript of R is the passenger that is replacing the super-scripted passenger and that the n^{th} passenger will now be following.

For R_{n-2}^{n-1}, there are two cases: $P_{n-1}^{n-2} = 1$ or 0, which means respectively the $(n-2)^{th}$ passenger is still in the aisle or has exited the aisle after the $(n-1)^{th}$ passenger exited the aisle. If $P_{n-1}^{n-2} = 1$,

$$\begin{aligned} R_{n-2}^{n-1} = & P_{static}^{n-2} \cdot T_{static}^{n-2} - (P_{static}^{n-1} \cdot T_{static}^{n-1}) + P_{arrival}^{n-2} \cdot (T_{interruption}^{n-2} + T_{luggage}^{n-2}) + \\ & R_{n-3}^{n-2} \cdot P_{arrival}^{n-2} - \frac{(Row^{n-2} - Row^{n-1}) \cdot D_{seat}}{V_{aisle}} \end{aligned} \tag{4.6}$$

Otherwise,

$$R_{n-2}^{n-1} = R_{n-3}^{n-1} \tag{4.7}$$

If the $(n-2)^{th}$ passenger has already exited the aisle ($P_{n-1}^{n-2} = 0$), then we will try to have the n^{th} passenger follow the $(n-3)^{th}$ passenger. If that does not work, we will call the recursive sequence again and again until we find a passenger that the n^{th} passenger can follow. However, if there is no passenger to follow in front of the n^{th} passenger, R would be zero.

When we make the replacement, we use the total time the $(n-2)^{th}$ passenger stops and subtract the previous terms that concerns the total time that the $(n-1)^{th}$ stops. Similarly, we will consider the case of $P_{arrival}^{n-2}$ by considering the interruption time of the $(n-2)^{th}$ passenger and R_{n-3}^{n-2}.

However, we realized that we may have over-counted: the time when the $(n-1)^{th}$ passenger is dealing with interruptions may occur simultaneously with the time when the $(n-2)^{th}$ passenger is dealing with arrival interruptions. Therefore, we write $\frac{(Row^{n-2}-Row^{n-1})\cdot D_{seat}}{V_{aisle}}$, which accounts for this and takes into consideration of all the different cases when the $(n-1)^{th}$ and $(n-2)^{th}$ passengers arrive.

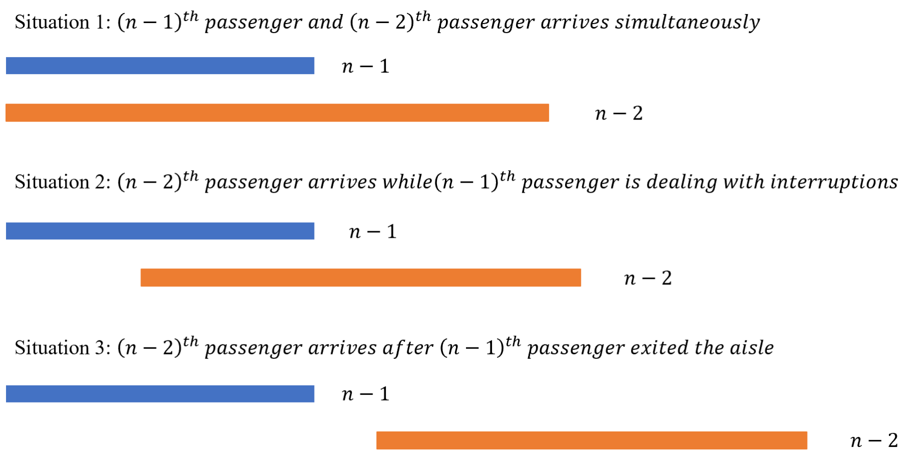


Figure 4.2: Different Cases of Consecutive Passenger Arrival Time. Bars for each passenger are their time spent dealing with interruptions.

4.3 Luggage Interruption Model

When a passenger meets luggage interruptions, we use our Luggage Interruptions Model to take the time delay that the n^{th} passenger would cause in the queue when they stop to deal with their luggage into consideration.

$$T_{luggage}^{n} = \underbrace{T_{put} \cdot N_{luggage}^{n}}_{\text{Time to put luggage in empty bin}} + \underbrace{N_{luggage}^{existing} \cdot \left(T_{arrange} + \beta \frac{N_{luggage}^{existing}}{N_{luggage}^{max}}\right)}_{\text{Time to re-organize the existing luggage}} \tag{4.8}$$

The total luggage interruptions time comes from the combination of time spent putting luggage in an empty bin and time spent re-organizing the bins to make space for the passenger's luggage if the bin was not empty.

The number of existing carry- on bags in a half-row of the N^{th} row (the overhead bin on one side of the aircraft is considered a half-row bin).

$$N_{luggage}^{existing} = \sum_{i=1}^{n-1} \alpha_i \cdot N_{luggage}^{i} \tag{4.9}$$

If the passenger before the n^{th} passenger has a desired row in the N^{th} row ($\alpha_i = 1$), then they would have stowed their luggage in the overhead bin, so the sum of the luggage from previous passengers is the number of existing luggage in the n^{th} passenger's overhead bin.

The maximum number of carry-on bags that passengers can put on the N^{th} half-row, with each passenger only allowed to bring two carry-on bags onto the aircraft.

$$N^{max}_{luggage} = 2 \cdot N_{passengers} \tag{4.10}$$

β is the amount of time it would take to re-arrange the entire half-row.

4.4 Seating Model - Including Seat Interruptions

When a passenger tries to be seated, they will have to deal with seating interruptions if any. Meanwhile in that process, when they at last exit the aisle, the passengers behind them can start walking in the aisle again. Therefore, we make two distinctions in the seating model: $T^n_{seating}$ and $T^n_{interruption}$.

$T^n_{seating}$ is the total time it takes for a passenger to deal with seating interruptions and be fully seated.

$$T^n_{seating} = V_{seat} \cdot N^{seat}_{grids} \tag{4.11}$$

We viewed the aircraft as a plane of grids. An aisle grid is one row in the aisle, and a seat grid is between each seat in a row.

N^{seat}_{grids} is the number of corresponding grids that the n^{th} passenger has to walk in order to be fully seated in the correct seat, and V_{seat} is the speed for a passenger to move between the two seat grids in a row.

$$V_{seat} = \alpha \cdot V_{aisle} \tag{4.12}$$

We set α to be 0.7 (in other words, 70% of the speed of moving across the aisle).

$T^n_{interruption}$ is the total time it takes for a passenger to deal with seating interruptions and leave the aisle so that they will not impact other passengers in the aisle.

$$T^n_{interruption} = V_{seat} \cdot N^{interruption}_{grids} \tag{4.13}$$

while $N^{interruption}_{grids}$ is the number of grids that the n^{th} passenger has to walk by in order to completely get out of the aisle (by completely, we mean that this passenger does not have to use the aisle again to deal with any interruptions).

Then, we consider all the different situations that could happen when a passenger arrives at their desired row. The table below lists N^{seat}_{grids} and $N^{interruption}_{grids}$ for every situation. A stands for the window seat, B stands for the middle seat, and C stands for the aisle seat.

Occupied Seats	Seat of Arrived Passenger	Grids to Walk to Not Be An Interruption	Grids to Walk to Be Properly Seated
NA	A,B, or C	1	A: 3; B:2; C:1
A	B or C	1	B:2; C:1
B	C	1	1
B	A	4	5
C	A or B	3	A: 4; B: 3
A and B	C	1	1
A and C	B	3	3
B and C	A	5	5

Table 4.1: Different Seating Situations and Their Interruption Grids and Seating Grids

5 Narrow Body Disembarking Time Model Based on Gantt Chart

The disembarking process is the reverse of the boarding process, so we use a similar bottom-up approach with the Gantt chart to model the disembarking process on a narrow body aircraft. Through calculating the single-passenger disembarking time, we generate a Gantt chart that will help us find the total disembarking time defined below:

Definition 5.1. *Total Disembarking Time: the time between when the first passenger starts walking off the plane and when the last passenger steps off the plane.*

The differences of boarding and disembarking process are:

- There are no seat interruptions in the disembarking process: the aisle passenger in a row has to exit first, and then the middle seat passenger, and finally the window seat passenger, so all disembarking methods have to be a combination of boarding by seat and another method.
- Row^n and Row^{n-1} may be far apart from each other, so we considered different cases for their locations. If $Row^{n-1} > Row^n$, then the n^{th} and $(n-1)^{th}$ passenger will exit their seat and into the aisle simultaneously. If $Row^{n-1} \le Row^n$, then the n^{th} passenger will wait until the $(n-1)^{th}$ passenger has walked to $Row^n - 1$, and then the n^{th} passenger will exit their seat and move into the aisle.
- If the $(n-1)^{th}$ passenger is in front of the n^{th} passenger by a large amount of space in the aisle, then we allow available passengers in the rows between the two passengers to exit into the aisle simultaneously.

Our main model is the single passenger disembarking time model, which looks the exact same as the single passenger boarding time model. Since we are not considering any seat interruptions, the seating model is simply the time it takes to get out of a seat and walk into the aisle.

$$T^n_{disembarking} = T^n_{d-seating} + T^n_{d-luggage} + T^n_{d-aisle} \tag{5.1}$$

Just like the BTM, we will number each passenger based on our disembarking method. The additional limitation is that in the same row, the number of the passenger in aisle seat > that of the passenger in middle seat > that of the passenger in window seat.

$$T^n_{d-aisle} = \frac{Row^n \cdot D_{seat}}{V_{aisle}} + P^{n-1}_{static} \cdot T^{n-1}_{static} + P^{n-1}_{d-arrival} \cdot T^{n-1}_{luggage} + RD^n_{n+B^{n-1}_n} \cdot P^{n-1}_{d-arrival} \tag{5.2}$$

We define the first two terms in the exact same way as the BTM. Then, since there will not be any seat interruptions for the $(n-1)^{th}$ passenger, they will only deal with a luggage interruption, as shown in the third term.

$P^{n-1}_{d-arrival}$ is defined as the exact opposite from the boarding model because the $(n-1)^{th}$ passenger's arrival is when they enter the aisle from their seat, and not when they leave the aisle from their seat.

$$P^{n-1}_{d-arrival} = \begin{cases} 1, \text{ if } Row^{n-1} > Row^n \\ 0, \text{ if } Row^{n-1} \le Row^n \end{cases} \tag{5.3}$$

The recursive sequence for the disembarking process is also different from the boarding process. First, we define B^{n-1}_n as the number of rows with at least one passenger between the n^{th} and $(n-1)^{th}$ passenger.

$$B_n^{n-1} = \begin{cases} \sum_{i=Row^{n-1}}^{Row^n} AV_i, \text{ if } Row^{n-1} > Row^n \\ 0, \text{ if } Row^{n-1} \leq Row^n \end{cases} \tag{5.4}$$

For every row, we want to find if there are available passengers: AV_i is a binary variable that determines whether there will be an available passenger in row i. If there are, those available passengers will exit into the aisle and would make the n^{th} passenger become the $(n + B_n^{n-1})^{th}$ passenger.

Now we consider the recursive sequence to make the n^{th} passenger become the $(n + B_n^{n-1})^{th}$ passenger. If $B_n^{n-1} \neq 0$,

$$RD^n_{n+B_n^{n-1}} = P_{static}^{n+B_n^{n-1}-1} \cdot T_{static}^{n+B_n^{n-1}-1} + P_{d-arrival}^{n+B_n^{n-1}-1} \cdot T_{luggage}^{n+B_n^{n-1}-1} - (P_{static}^{n-1} \cdot T_{static}^{n-1} + P_{d-arrival}^{n-1} \cdot T_{luggage}^{n-1}) \tag{5.5}$$

Otherwise, $RD^n_{n+B_n^{n-1}} = 0$. For this recursive replacement, the n^{th} passenger will now follow the motion of the $(n + B_n^{n-1} - 1)^{th}$ passenger instead of those of the $(n - 1)^{th}$ passenger.

The luggage interruptions model of the disembarking model is similar to that of the boarding model. When a passenger enters the aisle from their seat, they will have to take down their luggage (if any) from the overhead bins and deal with this interruption. The only variable that changes from Equation 4.8 is T_{put}. T_{put} is replaced by T_{take}, which is the time to take down one luggage from the overhead bins

The seating model takes how long it would take for each passenger to enter the aisle from their seat into consideration since there are no seating interruptions (no $T^n_{interruption}$). The equation for $T^n_{d-seating}$ is the same as $T^n_{seating}$, except the number of grids to move to get out of the seat is now different.

N_{grids}^{d-seat} takes on the following values for different types of seat: $N_{grids}^{d-seat} = 1, 2, 3$ for aisle seat, middle seat, and window seat, respectively because it is the number of grids between the seat and the aisle.

6 Monte Carlo Simulation and Results

The Monte Carlo simulation models the boarding and disembarking processes with the boarding and disembarking time calculated by our models, and we perform the simulations in Python. [9] The essence of our Monte Carlo simulation is generating a sequence of passengers, carry out every ”step” of their action with our models, and find the time of those steps. With the passengers' boarding and disembarking times altogether, we find the total boarding and disembarking times.

In generating the boarding sequence, every boarding method warrants a different way to generate, and boarding methods are partially randomized. After generating boarding sequence, we imitate passengers following each other in the aisle of the aircraft, and the delays they cause with their interruptions for each other.

In the disembarking process, a similar simulation happens so that we can model the passengers leaving the aircraft. Overall, our Monte Carlo simulation aims to provide the most logical and realistic interpretation of passenger movement on aircraft along with our models above.

Algorithm 1 Boarding Simulation Algorithm

Require: Input the size of the plane, the number of passengers, and boarding method
  while Passenger not in seat > 0 **do**
    for P_n in n **do**
      if $Position_n = Row_n$ **then**
        Record $T^n_{boarding} = T_{interruption} + T_{luggage}$
      else
        $Position_n$ + 0.11m
      end if
    end for
    Timer + 0.1s
  end while

Our boarding and disembarking models are based on Gantt charts, and through calculating the individual passenger boarding times, we use the Gantt chart to find the total boarding time. Figure 6.1 is an example of a Gantt chart that we generated for the random boarding method.

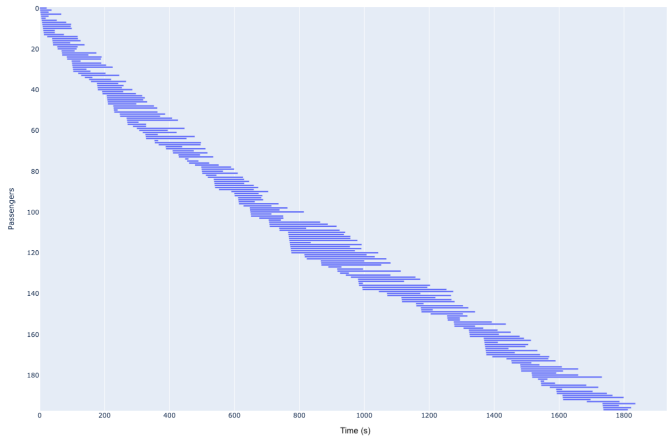


Figure 6.1: Example of Gantt Chart Modeling the Boarding Process

We can observe many different things going on in the Gantt chart. For example, at the top left of the plot, there were no interruptions in the first fourteen passengers: the gradual tilting suggests that each passenger boarded right after the one in front of them. The first interruption occurred after the boarding of passenger No.14, shown in the graph as bar No.15 is farther away from No.14. This can be explained by passenger No.6, who took 8 seconds to find their seat, allowing eight more passengers to board after him before he started putting his bag and stopping the passengers after him. Cases like this are easy to find in the Gantt chart, and we can understand the entire boarding process from the chart.

Five boarding methods are in the running to be evaluated as the best boarding method for the narrow body aircraft. Using our Monte Carlo simulation, we model the boarding and disembarking a hundred times in order to find the practical maximum, practical minimum, and average of the total boarding time of the different methods. We draw a boxplot to help us further understand the boarding methods' efficiency and some distribution graphs to help us visualize the distribution of boarding times.

	Random	Boarding by Section	Boarding by Seat	Reverse Pyramid	Steffen Method
Practical Maximum	2142.733	2193.838	1914.508	1639.61	573.595
Practical Minimum	1566.052	1572.383	1436.438	1324.71	530.972
Average	1850.918	1870.718	1642.778	1482.436	551.2884

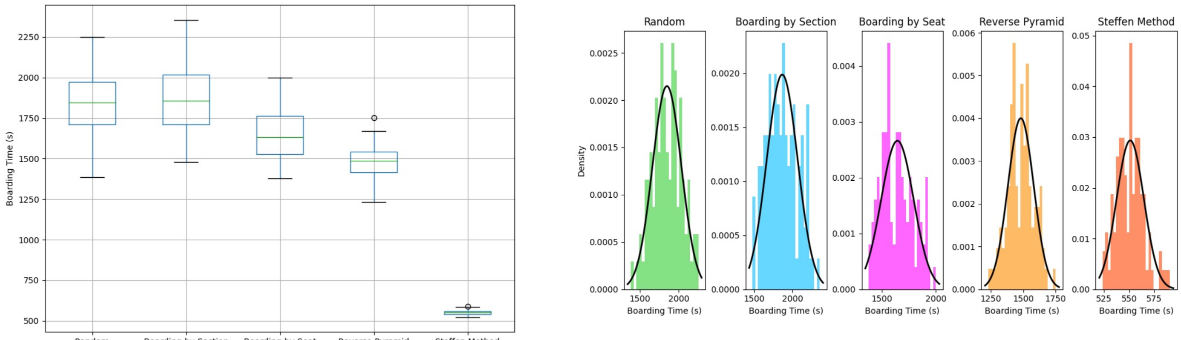


Figure 6.3: Boxplot and Distribution of Comparisons of Boarding Methods

As seen in Figure 6.3, all boarding methods' boarding time distributions follow a normal distribution, meaning that they are all stable and reliable methods.

From Figure 6.2 and 6.3, we conclude that the **optimal idealistic boarding strategy** is the **Steffen method**, as its practical maximum, practical minimum, and average boarding time are all lower than the rest of the methods. Furthermore, the time difference between the practical maximum and practical minimum is the smallest for the Steffen Method, which signifies that it is also very stable.

However, we also recognize that the Steffen method may not be the best boarding strategy for airlines because in order for the Steffen method to work, passengers will have to use a large amount of time making a queue based on their boarding number before they start the boarding process.

Therefore, we recommend the **reverse pyramid** boarding strategy as the **optimal practical boarding strategy**: it does not take a tremendous amount of time to make a queue before the boarding process, and it is expected boarding times are lower than other boarding methods. While boarding by section aims to minimize the luggage interruption and boarding by seat minimized the seat interruption time, reverse pyramid has the benefit of both, as it is a combination of the two methods.

Since all disembarking methods have to be boarding by seat because there may not be any seat interruptions, we only have three disembarking methods to choose from (note that they have to follow boarding by seat as well): Disembarking by Section, Reverse Pyramid, and the Steffen Method. These disembarking methods follow the same visualizations in Figure 2.2 except that the darker colors would board before the lighter ones and the larger numbers board first.

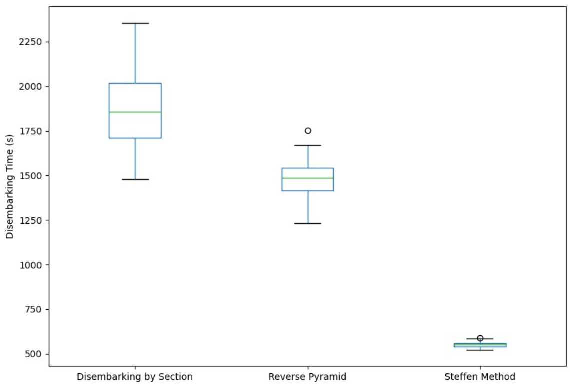


Figure 6.4: Boxplot Comparison of Different Disembarking Methods

As shown in Figure 6.4, we conclude that the **optimal practical disembarking method** is the **Steffen Method**. Although the Steffen Method was not practical for boarding, it could be implemented in real-life for disembarking because we don't have to make a long queue for disembarking - we just have to tell passengers who they will disembark after.

7 Sensitivity Analysis

We perform sensitivity analysis on our BTM to test how different parameters affect our models and results of the optimal boarding and disembarking strategies. We varied the following two parameters:

- The percentage of passengers not following the prescribed boarding or disembarking method.
- The average number of carry-on bags that passengers carry.

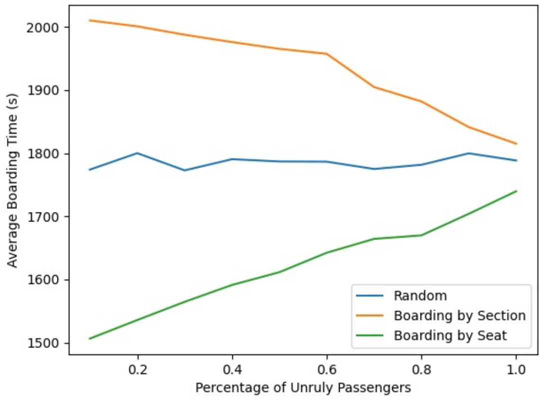


Figure 7.1: Effects of Unruly Passenger on the total boarding time

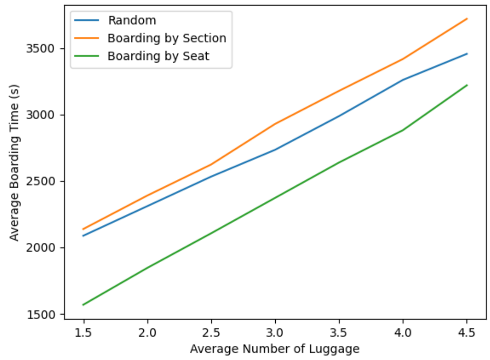


Figure 7.2: Effects of the Number of Passengers' Luggage on the total boarding time

As shown in Figure 7.1, we discover that as the percentage of unruly passengers increase, the performance of different methods gets closer to the one of random boarding, as the assignment of the passengers are getting more randomized. Since Boarding by Section takes longer to board than random in normal situation, its boarding time is getting better as it is becoming more identical with the random boarding method. Boarding by Seat experiences the opposite as it outperforms random at the start.

In Figure 7.2, we see a positive trend between the number of carry-on bags that passengers carry and the total boarding time. This is very reasonable because with passengers carrying more luggage, they will have to spend more time dealing with luggage interruptions.

In the case where passengers carry much more carry-on bags than normal and try to stow all their carry-on bags in the overhead bins, passengers may have to move to another row to complete this process, which means that when dealing with luggage interruptions, they will have to walk in reverse in the aisle. That, of course, will cause the total boarding time to increase, and it will complicate the boarding process even more.

Based on the sensitivity analysis, we can conclude that Boarding by Seat still outperforms the other two methods. In the two sensitivity analysis, Boarding by Seat performs the best comprehensively, as it is the most efficient in Figure 7.1 and is very close to being the most time-saving in Figure 7.2.

8 Model Applications

8.1 Flying Wing Models (FWM) Based on Queuing Networks

To model the boarding and disembarking process on a Flying Wing aircraft, we create a queuing network with our BTM and DTM bottom-up approach. In a queuing network, there are customers and servers; in our case, the passengers are the customers, the aisles are where the queue are located, and intersections between the aisles are where servers are.

In the Flying Wing aircraft, there are five aisles as seen in Figure 8.1: a top aisle and aisles A,B,C and D. When a passenger enters the Flying Wing aircraft, they will go to the top aisle first (we consider the entrance to be a part of the top aisle). Then based on their seat, the passenger will go to aisle A, B, C or D. In each of the intersection point between aisles A, B, C, or D and the top aisle, there is a server. If the passenger belongs in that aisle (A, B, C or D), then they will be taken in by that server from the top aisle, and now they will try to find their seat in the aisle A, B, C or D. However, if the passenger does not belong in that aisle, they will continue walking in the top aisle (if possible) until they reach their desired server. When disembarking, a similar situation occurs. Now, passengers go into the top aisle from aisles A, B, C or D. When a passenger arrives at an intersection, they will be randomly selected by the server to enter the top aisle, and they will be assigned a number in the queue there so that they will have a passenger to follow in the aisle.

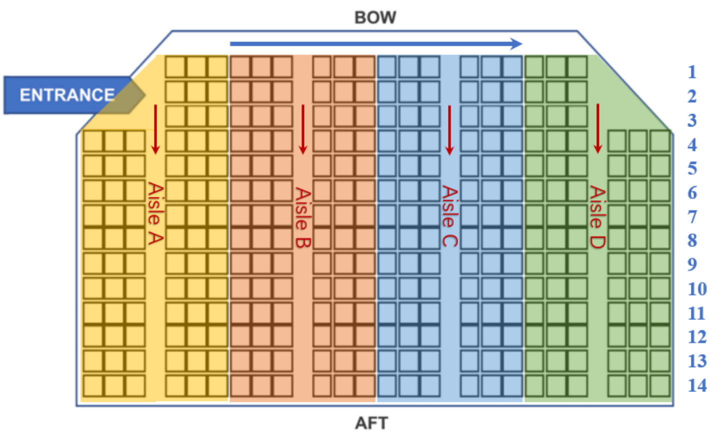


Figure 8.1: Different Aisles in the Flying Wing Aircraft

The single-passenger boarding and disembarking times are as follow:

$$T^n_{f-boarding} = T^n_{aisle} + T^n_{luggage} + T^n_{seating} + T^n_{top} \tag{8.1}$$

$$T^n_{f-disembarking} = T^n_{d-seating} + T^n_{d-luggage} + T^n_{d-aisle} + T^n_{d-top} \tag{8.2}$$

For each of the A, B, C, D aisles, we used our BTM and DTM, since they are nearly identical to the single aisle of the narrow-body passenger aircraft.

For the top aisle, passengers should know at the entrance (because of the flight attendants mentioned in the assumptions) which aisle (A, B, C, or D) they belong to. We model the passengers' motion in the top aisle with the following equations:

$$T^n_{top} = 6A^n_f \cdot V_{aisle} + P^{n-1}_{static} \cdot T^{n-1}_{static} + P^{n-1}_{arrival} \cdot P^{n-1}_{aisle} \cdot T^{n-1}_{aisle} + P^{n-1}_{arrival} \cdot RF^{n-1}_{n-2} \tag{8.3}$$

- A^n_f is the number of grids in the aisle that a passenger is away from the entrance after they have exited their aisle A, B, C, or D.

$$A^n_f = \begin{cases} 0, & \text{if passenger is in aisle A} \\ 1, & \text{if passenger is in aisle B} \\ 2, & \text{if passenger is in aisle C} \\ 3, & \text{if passenger is in aisle D} \end{cases} \tag{8.4}$$

- P^{n-1}_{aisle} is the binary variable that determines whether the aisle that the $(n-1)^{th}$ passenger has arrived at has a passenger at its entrance.
- T^{n-1}_{aisle} is the total time that it takes for the aisle that the $(n-1)^{th}$ passenger arrived at to clear out the passenger at its entrance (if that passenger moved on or if the passenger got seated).
- RF^{n-1}_{n-2} is the recurrence so that now the n^{th} passenger would be following the $(n-2)^{th}$ passenger instead of the $(n-1)^{th}$ passenger.

$$RF^{n-1}_{n-2} = \begin{cases} 0, \text{ if } n-2 \leq 0 \\ P^{n-2}_{static} \cdot T^{n-2}_{static} + P^{n-2}_{arrival} \cdot P^{n-2}_{aisle} \cdot T^{n-2}_{aisle} + P^{n-2}_{arrival} \cdot R^{n-2}_{n-3}, \text{ otherwise} \end{cases} \tag{8.5}$$

Through a similar fashion of how we derive the boarding aisle walking model for the FWM from the BTM, we design the disembarking aisle walking model for the FWM from the DTM.

$$T^{n}_{d-top} = 6A^{n}_{f} \cdot V_{aisle} + P^{n-1}_{static} \cdot T^{n-1}_{static} + RDF^{n-1}_{n} \cdot P^{n}_{new} \tag{8.6}$$

Passengers in aisle A, B, C, or D are randomly allowed into the top aisle once they arrive at the exit of those aisles. When they are randomly allowed in, the passenger behind them would have to wait and the passengers in the aisle behind the passenger coming in would be updated (e.g. the $(n)^{th}$ passenger would now become the $(n+1)^{th}$ passenger).

P^{n}_{new} is the possibility that a new n^{th} passenger is coming in from one of the aisles A, B, C, or D and making the original n^{th} passenger the $(n+1)^{th}$ passenger, and RDF^{n-1}_{n} is the recurrence that ensures that the n^{th} passenger, which has now became the $(n+1)^{th}$ passenger is following the new n^{th} passenger instead of the $(n-1)^{th}$ passenger. We define it just as the disembarking replacement recurrence.

Now that we have modeled each of the passengers' motion throughout the Flying Wing aircraft, we use Monte-Carlo simulations to find the total boarding and disembarking time.

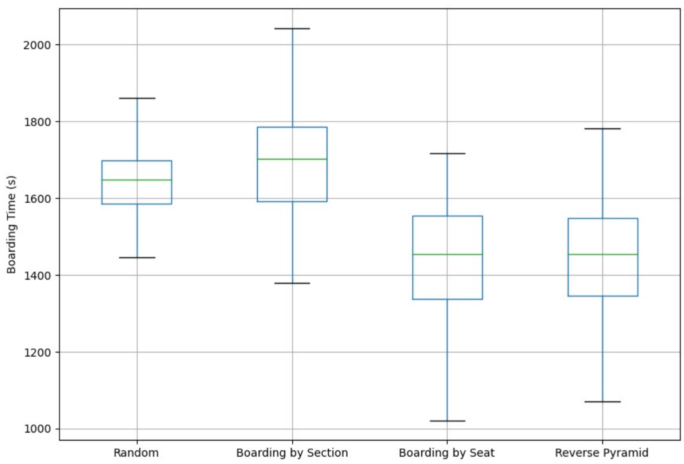


Figure 8.2: Boxplot Comparison of Different Boarding Methods for Flying Wing

We discarded the Steffen method in both Flying Wing Aircraft and Two-entrance, Two-aisle Aircraft because it is extremely impractical for the two aircraft considering they follow a queuing network. Among the remaining four methods, Boarding by Seat and Reverse Pyramid outperform the other two. We believe the reason of the tied performance between Boarding by Seat and Reverse Pyramid is the shortening of number of rows. Since there are less number of rows, the difference between Reverse Pyramid and Boarding by Seat is little to none as Reverse Pyramid differentiates itself by subdividing each column. Therefore, we conclude that Boarding by Seat is the best method for Flying Wing as it takes less time outside of the aircraft to form the line.

8.2 Two-entrance, Two-aisle Models (TTM) Based on Queuing Networks

Similar to our FWM, we use a queuing network for the Two-entrance, Two-aisle aircraft with changes to our BTM and DTM bottom-up approach. Similar to the FWM, we view the passengers as customers, aisles as where the queues are located, and intersections between the aisles to be the servers.

In the Two-entrance, Two-aisle aircraft, there are sixes aisles seen below in Figure 8.3. When a passenger's seat is covered by aisle A or B, they enter through the left entrance and aisle, and when a passenger's seat is covered by aisle C or D, they enter through the right entrance and aisle.

In the intersection points between the left aisle and aisle A or B and the intersection points between the right aisle and aisle C or D, the server will deal with a passenger if they belong to the aisle corresponding to the server. The server determines if the passenger can come into that aisle A, B, C, or D and how long it takes for the passenger to walk into that aisle and find their seat. The passenger will keep walking in the left or right aisle until they find their desired server. When disembarking, the same situation occurs, except that the entrance now becomes the exit.

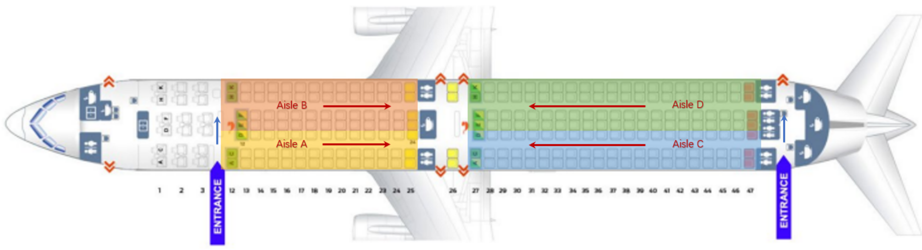


Figure 8.3: Different Aisles in the Two-entrance, Two-aisle Aircraft

The single-passenger boarding and disembarking times are as follow.

$$T^n_{t-boarding} = T^n_{aisle} + T^n_{luggage} + T^n_{seating} + T^n_{entrance-aisle} \tag{8.7}$$

$$T^n_{t-disembarking} = T^n_{d-seating} + T^n_{d-luggage} + T^n_{d-aisle} + T^n_{d-entrance-aisle} \tag{8.8}$$

For each of the A, B, C, D aisles, we implement our BTM and DTM, except for the seating model.

The seating model equation remains the same, but the number of grids to walk to not be an interruption, the number of grids to walk to be seated, and the number of grids to walk to exit the aisle are now different.

For aisles A and C, let A be the window seat, C be the seat next to the window seat, and D be the seat in the other aisle. Note that D will not be affected by A or C. N^{seat}_{grids}, $N^{interruption}_{grids}$, and N^{d-seat}_{grids} are redefined as follows.

Occupied Seats	Seat of Arrived Passenger	Grids to Walk to Not Be An Interruption	Grids to Walk to Be Properly Seated
NA	A or C	1	A: 2; C: 1
A with D and A without D	C	1	1
C with D and without D	A	1	3
Any combinations of A and C	D	1	1

Table 8.1: Different Seating Situations and Their Characteristics

For aisles B and D, let K be the window seat, H be the aisle seat. N^{seat}_{grids}, $N^{interruption}_{grids}$, and N^{d-seat}_{grids} are redefined as follows.

Occupied Seats	Seat of Arrived Passenger	Grids to Walk to Not Be An Interruption	Grids to Walk to Be Properly Seated
NA	K or H	1	K: 2; H: 1
K	H	1	1
H	K	1	3

Table 8.2: Different Seating Situations and Their Characteristics

For the entrance aisles (left or right aisle), passengers should know at the entrance which of aisles A, B, C,or D they belong to (because of the flight attendant mentioned in the assumptions). Therefore, we model the passengers' motion in the entrance aisles with the following equations:

$$T^n_{entrance-aisle} = A^n_t \cdot V_{aisle} + P^{n-1}_{static} \cdot T^{n-1}_{static} + P^{n-1}_{arrival} \cdot P^{n-1}_{aisle} \cdot T^{n-1}_{aisle} + P^{n-1}_{arrival} \cdot RF^{n-1}_{n-2} \tag{8.9}$$

Since the left and right aisles / entrance aisles of the Two-entrance, Two-aisle aircraft operates in the same way as the top aisle in the Flying Wing model, we use a similar boarding aisle walking model with the variables defined as the same. The only difference is that we replaced variable A^n_f with A^n_t.

A^n_t is the number of grids a passenger is away from the entrance after they have exited their aisle A, B, C, or D.

$$A^n_t = \begin{cases} 2, & \text{if passenger is in aisle A or C} \\ 6, & \text{if passenger is in aisle B or D} \end{cases} \tag{8.10}$$

Similarly, we use the same disembarking aisle walking model from the FWM, for the entrance aisles and the top aisle have the same functions. A^n_t is the only variable defined differently.

$$T^n_{d-entrance-aisle} = A^n_t \cdot V_{aisle} + P^{n-1}_{static} \cdot T^{n-1}_{static} + RDF^{n-1}_n \cdot P^n_{new} \tag{8.11}$$

Now that we have finished the model for each of the passengers' motion throughout the Two-entrance, Two-aisle aircraft, we use Monte-Carlo simulations to find the total boarding and disembarking times.

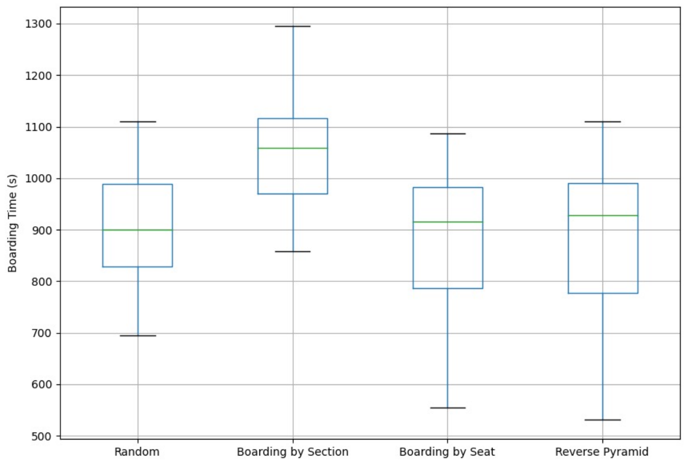


Figure 8.4: Boxplot Comparison of Different Boarding Methods for Two-Entrance Plane

Similar to the result of Flying Wing Aircraft, the performance of Boarding by Seat is tied with the one of Reverse Pyramid for Two-entrance, Two-aisle Aircraft. We believe the reasoning is similar as well–Reverse Pyramid is a combination of Boarding by Section and Boarding by Seat, either removing columns or removing rows will prevent Reverse Pyramid from making a differentiating boarding plan. In the case of the Two-entrance, Two-aisle Aircraft, the small aisle-to-column ratio causes Reverse Pyramid to make a very similar boarding plan as Boarding by Seat. Therefore, we conclude that **Boarding by Seat** is the best method for Two-entrance, Two-aisle aircraft.

8.3 Pandemic Boarding Models

In a pandemic situation where there is a capacity to the number of passengers, we adjust our previous boarding models in order to find the optimal boarding methods under this special circumstance. There must be social-distancing measures, so the distance between passengers will have to increase in the aisle and in the seats.

In the aisle, passengers will follow the same aisle walking models for the aircraft, but when performing the Monte Carlo Simulations, we will change the distance between the n^{th} and $(n-1)^{th}$ passengers based on the percentage of open seats. For the 30% situation, passengers will now have three grids of distance between each other; for the 50% situation, two grids of distance; for the 70% situation, one grid of distance.

In the seats of the narrow body aircraft, passengers will have to sit in the ways depicted in Figure 8.5.

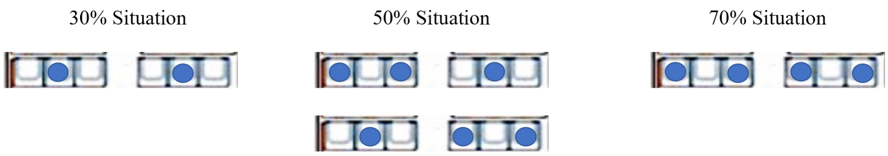
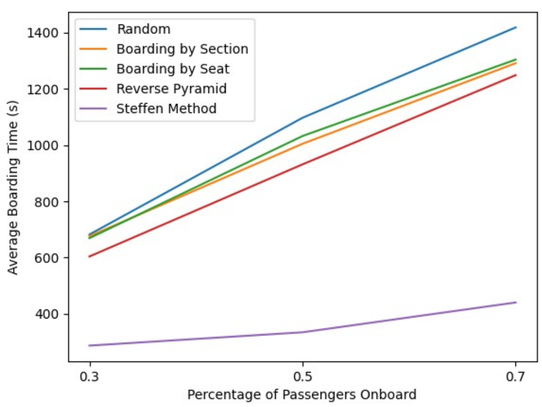


Figure 8.5: Seating for Different Pandemic Situations

Therefore, with the new seating arrangement, the number of grids to walk to not be an interruption, the number of grids to walk to be seated, and the number of grids to walk to exit the aisle will be different (N_{grids}^{d-seat}, $N_{grids}^{interruption}$, and $N_{grids}^{seating}$ are affected).

Through a similar fashion, we consider the boarding and disembarking process of the Flying Wing aircraft and the Two-entrance, Two-aisle aircraft under different pandemic situations.

We applied our models using the Monte Carlo Simulations and acquired the following results.



	Random	Boarding by Section	Boarding by Seat	Reverse Pyramid	Steffen Method
30%	681.775	675.79	668.8823	603.591	286.4
50%	1097.331	1004.6717	1032.3487	931.4895	333.7483
70%	1418.861	1291.5727	1304.3683	1249.0176	439.8503

Figure 8.6: Boarding Strategies Under Different Pandemic Situations for the Narrow-Body Aircraft

In the figure above, the **Steffen Method** shows itself to be the best under all pandemic situations, as its average boarding times are the smallest among the different boarding methods. However, as stated above, the Steffen Method is very idealistic for the boarding process. Therefore, we recommend **Reverse Pyramid** as the **optimal practical boarding method** under all pandemic situations. The recommended boarding methods under pandemic situations for the narrow-body aircraft does not differ from under normal situations.

9 Model Strengths and Shortness

Our models have several strengths:

- Our models are very comprehensive, as we consider nearly all processes that passengers undergo in an aircraft, and how each process contributes to the total boarding and disembarking time.
- We collected and incorporated data into our models for certain variables and made our models more realistic.
- Through mathematical equations and recurrences, we modeled the motion of each passenger in the boarding and disembarking process, which showed the practicality of our models.
- We applied our boarding and disembarking models to different situations and aircraft, showing that it can be generalized and is very applicable.

There are also a few areas of improvement for our models:

- In our boarding and disembarking models, we did not consider the acceleration of speed of passengers on the aircraft, and adding that to our model would make it more realistic.
- We did not consider the speed differences between passengers, and extremely slow or fast passengers could have altered our results.
- Despite having built pandemic boarding and disembarking models, we did not find the optimal disembarking method for the three aircraft under different pandemic situations.

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Appendix A Raw Seat Data for Gantt Chart

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Row	16	20	15	33	13	8	13	19	27	25	22	7	6	10	29	25	25	8
Seat	2	6	6	2	4	1	3	3	2	1	1	1	6	3	3	4	2	4
Luggage #	1	2	1	2	1	1	2	2	1	1	2	1	1	1	1	2	1	2

	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Row	21	15	11	10	33	12	30	28	6	21	23	31	4	4	12	30	2	12
Seat	4	2	3	5	2	1	2	5	6	1	2	1	1	4	6	4	6	2
Luggage #	2	2	2	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1

	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
Row	32	17	20	18	21	15	19	31	33	27	27	14	28	29	1	24	29	23
Seat	6	6	5	4	3	3	6	5	4	4	6	3	1	3	3	2	3	6
Luggage #	2	2	1	1	2	1	2	1	1	2	2	2	2	1	2	2	2	1

	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
Row	28	33	3	3	2	25	9	10	4	29	13	1	27	19	5	24	12	22
Seat	3	6	2	1	6	3	2	6	2	3	5	2	3	1	3	6	6	5
Luggage #	2	2	1	1	2	2	1	2	2	1	1	1	2	1	2	2	2	1

	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Row	12	19	1	3	7	17	24	24	15	17	6	16	13	26	10	10	26	8
Seat	5	5	5	2	5	2	1	3	4	5	4	4	3	6	4	1	2	5
Luggage #	1	2	1	1	1	1	1	1	2	1	1	2	2	1	2	1	2	1

	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
Row	33	23	24	22	9	7	29	11	22	9	31	6	9	7	5	26	28	28
Seat	2	2	5	6	1	5	4	6	3	1	1	2	4	5	6	1	3	2
Luggage #	2	1	1	1	2	2	2	1	2	1	2	1	2	1	1	2	2	2

	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126
Row	14	23	30	25	27	26	30	8	32	14	26	16	29	24	23	30	18	20
Seat	5	5	4	5	3	6	6	2	2	1	4	3	1	3	1	6	3	1
Luggage #	1	1	1	1	1	2	2	2	1	1	1	2	1	1	2	1	1	2

	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144
Row	17	3	6	23	1	11	18	20	11	1	32	15	25	33	6	28	11	16
Seat	3	5	1	1	2	2	1	4	5	3	5	5	1	5	3	4	5	6
Luggage #	1	1	1	1	1	1	2	1	1	1	1	2	1	1	2	1	2	1

	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162
Row	12	4	31	31	2	32	15	9	7	3	19	22	5	4	16	18	10	26
Seat	5	4	6	2	4	6	3	1	6	2	4	1	5	1	4	5	1	6
Luggage #	2	1	1	1	1	2	2	1	1	2	1	1	1	1	2	2	2	2

	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
Row	16	30	7	20	18	5	17	5	14	27	18	21	4	3	20	22	13	11
Seat	1	5	2	6	4	6	5	3	4	4	6	2	1	3	2	3	1	4
Luggage #	1	1	1	2	1	1	2	1	1	1	2	1	1	2	2	2	1	2

	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198
Row	17	32	5	2	2	8	21	2	9	14	21	31	1	8	32	14	19	13
Seat	5	3	2	4	4	3	2	4	4	6	6	2	5	1	4	5	5	4
Luggage #	1	1	2	1	2	1	2	1	2	1	1	2	2	1	2	1	1	2