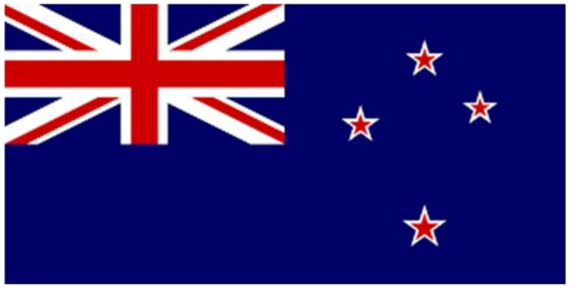
Developing an Annual Scheduling model for a Global Sporting League

Macleans College

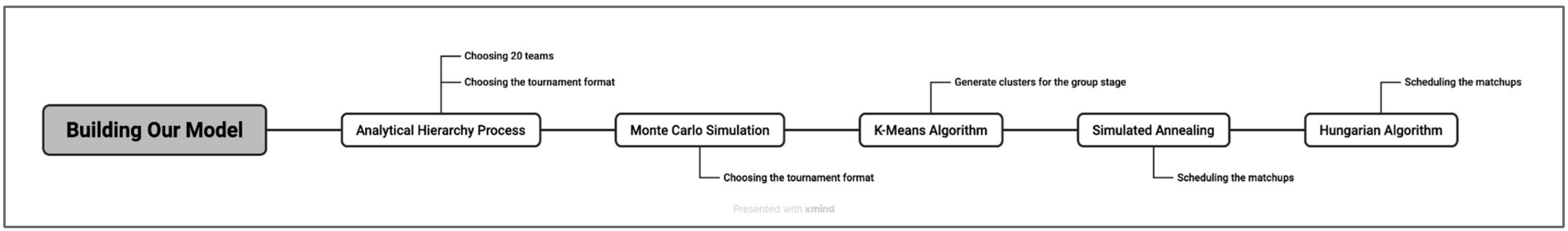


Introduction

- Traditionally, sports leagues are **regional tournaments**.

- But we're developing a model for the **selection of teams** and the **scheduling of matches** on a global scale.
    - This model should also be able to be adaptable to different sports and different tournament sizes.

- Our sport of choice is **football**, and the first Global Sports League tournament will happen in 2027.


Building Our Model
Analytical Hierarchy Process
Choosing 20 teams
Choosing the tournament format
Monte Carlo Simulation
Choosing the tournament format
K-Means Algorithm
Generate clusters for the group stage
Simulated Annealing
Scheduling the matchups
Hungarian Algorithm
Scheduling the matchups
Presented with xmind

Key considerations and concepts

Additional Considerations:

- Environmental Impact
- Time Zones - Athletes and Fans
- Athlete Rests
- Number of games played
- Home/Away advantage

Key Concepts

What makes a scheduling system fair and balanced?

- Team rankings at the end of the tournament align with their actual skill
- Equal travel times and number of games
- Balanced competitive matchups

Aspects that affect athlete performance

- Differences in time zones
- Home game advantage

Factors that promote environmental sustainability

- green energy
- travel times and the distance of flights

Fan engagement

- all teams have at least one home game
- across time zones, fans are able to watch at reasonable times

Our model consists of three parts:

1. Quantitative analysis of all countries - **Analytical Hierarchy Process**
    a. Global standings
    b. Internal fan support
    c. Potential for Growth
    d. Environmental harm
2. Deciding for the tournament format - **Monte Carlo Method**
    a. Travel time
    b. Number of matches played by each team
    c. Accuracy of the final standings by the end of each tournament.
    d. Many formats are grouped -> we use a **K-means Clustering Algorithm**.
3. Modelling the ideal scheduling of games - **Simulated Annealing**
    a. The order of games
    b. Athlete performance
    c. Home/away games
    d. Climate
    e. Time of day to play a match

Part 1: Defining and selecting a sport

Definition of sport

We will only consider **physical sports** due to:

- Geographical diversity
- Global sporting leagues promote physical activity
- Cultural importance of physical sports - widely recognised

Selection of sport

Our sport will be **Association Football (Soccer).**

- Large number of fans (3.5 billion)
- It is a highly accessible sport, very popular in regions of developing countries
  - With this, we know our GSL will work better on a global scale.

Part 1: Team selection criteria and aspects considered

1. Current **World Football Elo Ratings**
   Justification: The Elo rating system provides an accurate measure of a team's skill. A large league like the GSL will need similar skilled teams to maximise revenue which is vital for the league to be sustainable.

1. Total **number and proportion of football fans**
   Justification: This is a measure for the cultural effect that the sport has on each country. This way, we prioritise countries that would derive the most enjoyment from the tournament.

1. **Government sports funding** as a percentage of GDP
   Justification: Countries with high amounts of sports funding are likely to have good infrastructure to host these games well, as well as having more youth development to promote the future growth of the program.

1. Proportion of **renewable energy production** of a country
   Justification: Countries involved in the tournament will likely expect to host games. However, the operation of major stadiums and sports games leads to significant carbon emissions and environmental impact.

Each of these criteria are adaptable for each different sports. E.g. to replace the **World Football Elo Rating** would be other credible rating systems that rank a country.

Part 1: AHP to generate priority index

- 5 uniformly distributed 0-1 indexes.
    - Skill Index -> Elo
    - Fan Proportion Index
    - Fan Count Index
    - Government Funding Index
    - Green Index

Skill Index and Fan Proportion Index -> min-max normalization
Fan Count Index and Government Index -> z-score normalisation
Green Index -> Already a nice uniform distribution.

AHP Gives weights (2.84, 0.78, 0.62, 0.33, 1)

Part 1: AHP

1	3	5	7	9	2,4,6,8
Equal Importance	Slightly More Important	Significantly More Important	Strongly More Important	Extremely More Important	Values In Between

	SKILL	FAN PROP	FAN COUNT	FUNDING	RENEWABLE
SKILL	1	4	4	8	6
FAN PROP	1/4	1	2	5	1/3
FAN COUNT	1/4	1/2	1	5	1/3
FUNDING	1/8	1/5	1/5	1	1
RENEWABLE	1/6	3	3	1	1

$$I_{priority} = 2.84 \cdot I_{skill} + 0.78 \cdot I_{fan\ proportion} + 0.63 \cdot I_{fan\ count} + 0.333 \cdot I_{funding} + 1 \cdot I_{green}$$

Part 1: 20 teams

In accordance with our goals for team selection, we select the two highest priority teams from each continent to fulfill geographical representation, and then the remaining 12 countries are selected based on their **priority index.**

The 20 chosen countries are:
Brazil, Spain, France, Argentina, Uruguay, Colombia, England, Paraguay, Germany, Ecuador, Portugal, Italy, Morocco, Egypt, South Korea, Japan, Mexico, Costa Rica, New Zealand and Australia.

Part 2: Deciding format

We need a tournament format for the GSL.

We consider:

- Total games played
- Total airtime travel
- Standard deviation of number of games played for each team
- Standard deviation of airtime travelled for each team
- Accuracy of our format

With an AHP, The most appropriate weights for these factors are: (0.21, 0.34, 0.07, 0.10, 0.28).

Tournament Design	
T_g	Total number of games played in a season (lower = better)
T_a	Total air time of travels for all the teams within the season (lower = better)
$N_1, N_2, \cdots, N_{20}$	Number of games that each individual team play in the season
σ_N	Standard deviation of the dataset $(N_1, N_2, \cdots, N_{20})$ (lower = better)
$A_1, A_2, \cdots, A_{20}$	Total air time for each individual teams through the season
σ_A	Standard deviation of the dataset $(A_1, A_2, \cdots, A_{20})$ (lower = better)
P_A	$\frac{\sigma_A}{T_a}$ (lower = better)
L	Loss index: The size of difference between the expected ranking (following our criteria for fairness) and the outcomes caused by each format (lower = better)
I_F	Format Adequacy Index: how suitable, overall, a tournament format is for the selected twenty countries. (higher = better)
Mapping and Scheduling	
φ	Latitude
λ	Longitude

	T_g	T_a	σ_N	P_A	L
T_g	1	1/2	5	2	1/2
T_a	2	1	5	5	1/2
σ_N	1/5	1/5	1	1	1/3
P_A	1/2	1/5	1	1	1/2
L	2	2	3	2	1

Part 2: tournament formats

- **#1 Full Single Round Robin**
  - Each team plays every other team once, and are ranked based on their accumulated points.
- **#2 Full Double Round Robin**
  - Each team plays every other team twice, and are ranked based on their accumulated points.
- **#3 Hybrid Grouped Single Round Robin + Knockouts (by region)**
  - Teams are allocated groups based on their region and play each other once. The best teams progress to a knockout stage.
- **#4 Hybrid Grouped Double Round Robin + Knockouts (by region)**
  - Teams are allocated groups based on their region and play each other twice. The best teams progress to a knockout stage.
- **#5 Hybrid Grouped Single Round Robin + Knockouts - (by region + size)**
  - Teams are allocated groups based on their region and play each other once. Larger regions are split up so all groups are of similar size. The best teams progress to a knockout stage.
- **#6 Hybrid Grouped Double Round Robin + Knockouts - (by region + size)**
  - Teams are allocated groups based on their region and play each other twice. Larger regions are split up so all groups are of similar size. The best teams progress to a knockout stage.
- **#7 Random Knockout**
  - A knockout bracket is generated and teams play until they are knocked out.

Part 2: tournament formats

- **Full Single Round Robin**
  - All teams are placed in a combined pool, playing every other team exactly once. The teams are then ranked according to the '3 points for a win' scoring system, and that becomes the result of the tournament.
- **Full Double Round Robin**
  - All teams are placed in a combined pool, playing every other team twice, one home and one away. The teams are then ranked according to the '3 points for a win' scoring system, and that becomes the result of the tournament.
- **Hybrid Grouped Single Round Robin + Knockouts (by region)**
  - Teams are separated into 3 groups of 4 teams and 1 group of 8 teams based on their geographical location and proximity to other national teams. A single round robin is then played within each group. Based on the '3 points for a win' scoring system, 1st place in each of the 4 groups will advance to the knockout stage. 2nd and 3rd in each group will advance to the qualifier single round robin group to determine the top 4 teams to join the knockout stage. Teams below 3rd in each group will be placed into the placement single round robin group, and their final rank will be determined by points accumulated in the placement single round robin. The knockout bracket will be paired in a way so that the first seed (based on elo) versus the eighth seed, the second seed versus the seventh seed and so on. The final standings will be as follows: knockout winner, knockout runner up, semifinalists ranked by seed, quarterfinalists ranked by seed, qualifier single round robin teams ranked by points, and placement single round robin teams ranked by points.
- **Hybrid Grouped Double Round Robin + Knockouts - (by region)**
  - Teams are separated into 3 groups of 4 teams and 1 group of 8 teams based on their geographical location and proximity to other national teams. A double round robin is then played within each group so that each national team plays each other once at home and once away. Based on the '3 points for a win' scoring system, 1st place in each of the 4 groups will advance to the knockout stage. 2nd and 3rd in each group will advance to the qualifier double round robin group to determine the top 4 teams to join the knockout stage. Teams below 3rd in their group stage will be placed into the placement double round robin group, and their final rank will be determined by points accumulated in the placement double round robin. The knockout bracket will be paired in a way so that the first seed (based on elo) versus the eighth seed, the second seed versus the seventh seed and so on. The final standings will be as follows: knockout winner, knockout runner up, semifinalists ranked by seed, quarterfinalists ranked by seed, qualifier double round robin teams ranked by points, and placement double round robin teams ranked by points.
- **Hybrid Grouped Single Round Robin + Knockouts - (by region + size)**
  - Teams are separated into 5 groups of 4 teams based on their geographical location and proximity to other national teams. A single round robin is then played within each group. Based on the '3 points for a win' scoring system, 1st place in each of the 5 groups will advance to the knockout stage. 2nd and 3rd in each group will advance to the qualifier single round robin group to determine the top 3 teams to join the knockout stage. Teams below 3rd in each group will be placed into the placement single round robin group, and their final rank will be determined by points accumulated in the placement single round robin. The knockout bracket will be paired in a way so that the first seed (based on elo) versus the eighth seed, the second seed versus the seventh seed and so on. The final standings will be as follows: knockout winner, knockout runner up, semifinalists ranked by seed, quarterfinalists ranked by seed, qualifier single round robin teams ranked by points, and placement single round robin teams ranked by points.
- **Hybrid Grouped Double Round Robin + Knockouts - (by region + size)**
  - Teams are separated into 5 groups of 4 teams based on their geographical location and proximity to other national teams. A double round robin is then played within each group so that each national team plays each other once at home and once away. Based on the '3 points for a win' scoring system, 1st place in each of the 5 groups will advance to the knockout stage. 2nd and 3rd in each group will advance to the qualifier double round robin group to determine the top 3 teams to join the knockout stage. Teams below 3rd in their group will be placed into the placement double round robin group, and their final rank will be determined by points accumulated in the placement double round robin. The knockout bracket will be paired in a way so that the first seed (based on elo) versus the eighth seed, the second seed versus the seventh seed and so on. The final standings will be as follows: knockout winner, knockout runner up, semifinalists ranked by seed, quarterfinalists ranked by seed, qualifier double round robin teams ranked by points, and placement double round robin teams ranked by points.
- **Random Knockout**
  - Teams are placed into a knockout bracket by random, and if they lose a match, they are eliminated from the tournament (due to the fact we have 20 teams which isn't a power of 2, there will be teams with BYEs). The final rankings will be as follows: knockout winner, runner-up, and then the remaining teams based on highest round reached (eg. semifinals) and then team seed.
- **Expanded Hybrid Grouped Single Round Robin + Knockouts - (by region + size)**
  - Teams are separated into 6 groups of 4 based on their geographical location and proximity to other national teams. A single round robin is then played within each group. Based on the '3 points for a win' scoring system, 1st place in each of the 6 groups will advance to the knockout stage. 2nd and 3rd in each group will advance to the qualifier single round robin group to determine the top 2 teams to join the knockout stage. Teams below 3rd will be placed into the placement double round robin group, and their final rank will be determined by points accumulated in the placement single round robin. The knockout bracket will be paired in a way so that the first seed (based on elo) versus the eighth seed, the second seed versus the seventh seed and so on. The final standings will be as follows: knockout winner, knockout runner up, semifinalists ranked by seed, quarterfinalists ranked by seed, qualifier single round robin teams ranked by points, and placement single round robin teams ranked by points.

Part 2: Simulating tournament formats

We run each format 1000 times, then collect:

- Number of games played in each simulation
- Average combined airtime across all simulations
- Average ranking for each team across all simulations
- Average number of games played for each team
- Average standard deviation for the total flight times across each team
- Average loss (calculated later)

Functions used:

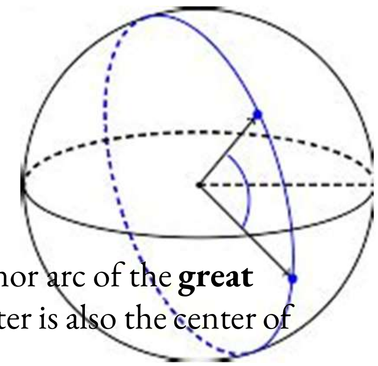
- win rate of countries
- distance between two countries
- evaluation for the loss index (L)
- simulating one match.

Part 2: Winrate function

Winrate: to calculate the win rate of one particular country over another, we deploy the **Elo** system
Where R_A, R_B are the elo ratings of teams A and B respectively.

$$P(A\ beats\ B) = \frac{1}{1 + 10^{(R_B - R_A)/400}}$$

Part 2: Distance between two countries function



- As geometrically the earth is a sphere, the shortest distance between two points on earth P and Q would be the minor arc of the **great circle** passing through the two points, that is the minor arc of the unique circle on the surface of earth, whose center is also the center of earth.
- The shortest distance between two points **P** and **Q** is then given by **D = ΘR/2** where is the angle **∠POQ** (where O is the centre of the earth) measured in radians and R = 6,371 km is the radius of the earth. Although the airplane does not sit on the surface of the earth, the maximum elevation a commercial of an airplane above ground 11km is insignificant and hence ignored in the calculation. Also the variation in the radius of the earth is not important.
- The **haversine formula** which states that $$\sin^2\left(\frac{\theta}{2}\right) = a = \sin^2\left(\frac{\varphi_P - \varphi_Q}{2}\right) + \cos(\varphi_P)\cos(\varphi_Q)\sin^2\left(\frac{\lambda_P - \lambda_Q}{2}\right)$$
- Where φ and λ represent correspond to the latitude and longitude of the points P and Q can then be used to find Θ.
- Keep in mind that latitude and longitude are both measured in degrees and should be first converted into radians. We first calculate a then rearrange to give

$$\tan^2\left(\frac{\theta}{2}\right) = \frac{\sin^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right)} \quad \left(\frac{\theta}{2}\right) = \frac{a}{1-a}$$

- Then to ensure that our value is the angle of the smaller arc we must keep the value of 2 in the first quadrant of the cartesian plane, we use $\arctan_2$ where both the inputted x and y coordinates are positive.

$$\theta = 2 \cdot \arctan_2\left(\sqrt{a}, \sqrt{1 - a}\right)$$

Part 2: Function for Evaluation for the loss index

- For every simulated tournament, to evaluate how accurate the results generated were, we developed a **loss score** that is based on the number of **inversions** in the ranking: that is the number of ordered pairs of teams A and B where A ended up ranking higher than B but B is actually better than A. Additionally we also weigh the significance of each inversion to account for the following factors:
    - It is generally less surprising when the ranking of teams who are close in Elo are inverted, compared to large differences in Elo because we would expect teams of similar Elo to be of similar skill and occasionally beat one another. However, when there is a large difference between Elo and hence skill, it is extremely unlikely that the less skilled team beats the more skilled one.
    - It is generally less significant when there are disparities between expected ranking and actual ranking when a team isn't ranked highly in both. This is because fans and viewers are usually most engaged with teams at the top of the ladder, and also because there is generally less to lose for a team who is ranked lower than they should have been if their original ranking was low to start.

  To account for the following factors, for each inversion we calculate the amount of **bias** in favour of one team winning compared to random chance. The more likely a team was to rank higher than another team, the more surprising it is if they rank lower. This is given by $Bias = |P(A\ beats\ B)\ -\ 50\%|$

- We also calculate a weight representing how significant the team is due to their high ranking. The significance of a particular ranking is inversely proportional to the rank itself. We calculate a combined weight score using $Weight = \frac{1}{Pr(Team\ 1)} + \frac{1}{Ar(Team\ 1)} + \frac{1}{Pr(Team\ 2)} + \frac{1}{Ar(Team\ 2)}$
- for a single inversion where Pr and Ar stand for a team's predicted rank and actual rank. We form this weight score using the principle that highly ranked team have more "weight" compared to lowly ranked and that the difference between high ranks such a #1 and #2 are more significant than the differences between #8 and #9. The reciprocal relationship is given by Zipf's Law which relates to the linguistic and psychological prioritisation for highly ranked items.
- The **loss score** of a single inversion is then given by taking the product of the **bias** and the **weight** of the inversion. The total **loss score** for each simulation is found by summing up the **loss score** of all inversions in the simulation

Part 2: Simulating one particular match

- To determine the rankings, we will follow the three points for a win standard (Keep the Score, 2025) , where we award the winning team with 3 points, and the losing team with 0 points. A random real value between 0 and 1 is generated, and if this value is less than the win rate of team A (given by the winrate function), we declare team A the winner, and conversely if it's larger, team B will be the winner.

Part 2: Tournament format comparison after running model

Format	T_g	T_a	σ_N	P_A	L	I_F
1	190 Games	3783 Hours	0.00	65.9 Hours	1.48	0.704
2	380 Games	7567 Hours	0.00	110.2 Hours	0.81	0.449
3	109 Games	1524 Hours	2.49	45.2 Hours	5.58	0.634
4	211 Games	2920 Hours	5.39	82.0 Hours	4.41	0.527
5	92 Games	1464 Hours	0.932	51.0 Hours	3.87	0.721
6	177 Games	2798 Hours	2.73	94.9 Hours	3.02	0.620
7	19 Games	354 Hours	0.52	19.3 Hours	8.03	0.617

Therefore we can see that, comparing the value of I_F across all different formats, we should choose **#5 Hybrid Grouped Single Round Robin + Knockouts - (by region + size)** as our tournament format. This has been highlighted above.

Part 2: Average number of games

On average, teams play:

Average number of games = 2 * Total number of games played/Number of countries in the tournament

But to be called champion you need at least 6 and at most 15 games to be called champion.

Average of 9.2 games for each team

- 1 game per month for each team

Part 2: Group stage pools selected by k-means clustering

1. Initialisation (Forgy Method): randomly choose k countries out of the 20 as the starting centroids.
2. Cluster each of the countries to their nearest centroid.
3. Calculate the centroid of each of our defined clusters as $P =((\varphi_1 + \varphi_2 + \ldots + \varphi_N)/N, (\lambda_1 + \lambda_2 + \ldots + \lambda_N)/N)$
4. Update the centroids list to our newly calculated centroids.
5. Repeat steps 2, 3 and 4 until the centroids from the previous iteration are exactly the same as the newly calculated centroids.

Part 2: k-means clustering example

As an example, below is the algorithm process for grouping the 20 countries into 3 clusters of 7, 7, and 6 countries. The solid circles represent the centroid of the clusters of that corresponding colour.

Part 2: Calculating the optimal number of clusters

So we have these clusters. Then we'll run the simulation 1000 times again to determine which one is the best.

No. of Clusters	2	3	4	5	6	7	8
I_F	0.436	0.445	0.637	**0.727**	0.650	0.452	0.501

Part 2: Final Clusters of countries in our group round robin

Part 3: Matchmaking and game locations for the schedule

In a single round robin tournament with n team each team has to play a total of n-1 separate games. We first create a schedule S where S[r][t] represents the location that each team t has to play at in round r.

We want to find a schedule which minimizes the total distance that each team will have to travel by plane. This is also going to be the most environmentally friendly schedule.

We also have some constraints that we must adhere to:

Part 3: Constraints to schedules

- Restrict max number of home games for each team
  - Teams play better on home turf..
- Teams should not have three consecutive matches where there is a significant difference in time zone between two matches.
- Teams should not have more than two consecutive matches where they are against teams which are much more skilled. We have decided that a team is considered significantly more skilled than another team if one team's odds of winning are **3:1** or more.

Hence we consider games to be heavily one sided if the difference in elo is greater than 190.

$$0.75 \ < \ P(A\, beats\, B) \ = \ \frac{1}{1+10^{\frac{1}{400}(R_B - R_A)}}$$

$$\Rightarrow \ R_B - R_A < 400 \ \log_{10}\left(\frac{1}{0.75} - 1\right) = \ 190$$

Part 3: NP hard problem and simulated annealing

As the tournament scheduling is an **NP hard** problem (Bhattacharyya, 2016) it is computationally difficult to find the optimal tournament schedule when the number of teams becomes large as in the case of the ten team knockout qualifier round robin tournament. In order to accommodate for this limitation and the possibility of expansion and generalization, we have decided to use **simulated annealing**: a **metaheuristic** algorithm which is able to efficiently find approximate optimal solutions for optimization problem, theses approximate solutions are good enough as the difference from the theoretically most optimal solution is marginal.
The algorithm explores the total space of all possible schedules by making small adjustments to the schedule over time. Initially the algorithm is given more freedom to make any adjustment randomly in order to incentivise it to explore the total state space so that it finds a solution close to theoretically the most optimal solution. Then over time the algorithm is constrained to only making adjustments that are more optimal so it settles towards the global maximum.

We do not want to prevent our algorithm from exploring schedules that violate our constraints as those can lead to more optimal solutions when modified slightly; only those that violate many constraints and cannot be fixed without drastic changes to the schedule. The cost function we use for our **simulated annealing** algorithm is given by:

$$cost(S) = \sqrt{[total\ distance(S)]^2 + (\omega \cdot f[number\ of\ constraint\ violations(S)])^2}$$

Where ω is a parameter of the **simulated annealing** algorithm and ***f(n)*** = ***1*** + ***√v*** * ***ln(√v)***.

Part 3: How does the algorithm work

Our algorithm for determining an optimal schedule works roughly as such:

1. Creating an initial schedule which is not optimal
2. Choose a random adjustment to make to the schedule
3. We decide whether or not to make the the adjustment (performed some number of times):
    - If the cost of the schedule after making the adjustment is less, we make the adjustment
    - Otherwise we choose to make the adjustment with probability $\mathbf{e^{(-\Delta cost\ /\ T)}}$ where **T** is the current temperature, which is a variable that determines how much freedom our algorithm is to make suboptimal adjustments.
1. We decrease the temperature **T** by a factor of **β: T ← T * β** where **β** is another parameter of our algorithm
2. Repeat steps 2, 3, and 4 some number of times.
3. Choose a schedule with zero violations and with the least cost out of all the schedules we have explored

Group	Asian Pacific	Central American	South American	Northern Europe	Mediterranean
Round 1	South Korea v. **Australia** *8pm GMT+10* **Japan** v. New Zealand *6pm GMT+9*	**Mexico** v. Ecuador *7pm GMT-6* Costa Rica v. **Colombia** *7pm GMT-5*	**Brazil** v. Argentina *7pm GMT-3* Paraguay v. **Uruguay** *7pm GMT-3*	**England** v. Germany *7pm GMT+0* **France** v. Spain *7pm GMT+2*	**Italy** v. Portugal *7pm GMT+2* **Morocco** v. Egypt *6pm GMT+1*
Round 2	**South Korea** v. New Zealand *6pm GMT+9* **Japan** v. Australia *7pm GMT+9*	Mexico v. **Colombia** *7pm GMT-5* **Costa Rica** v. Ecuador *7pm GMT-6*	Brazil v. **Uruguay** *7pm GMT-3* **Argentina** v. Paraguay *7pm GMT-3*	England v. **France** *7pm GMT+2* **Germany** v. Spain *7pm GMT+2*	**Italy** v. Morocco *7pm GMT+2* **Portugal** v. Egypt *6pm GMT+1*
Round 3	**New Zealand** v. Australia *7pm GMT+12* **South Korea** v. Japan *7pm GMT+9*	**Ecuador** v. Colombia *7pm GMT-5* **Costa Rica** v. Mexico *8pm GMT-6*	Uruguay v. **Argentina** *7pm GMT-3* **Paraguay** v. Brazil *7pm GMT-3*	France v. **Germany** *7pm GMT+2* **Spain** v. England *7pm GMT+2*	Italy v. **Egypt** *7pm GMT+2* **Portugal** v. Morocco *7pm GMT+1*

Part 3: Optimal temperatures

There are seasonal factors that impact whether or not it is suitable to play games in a particular time of year. The summer and winter months are particularly bad because of extreme temperatures and weather. Because of this we will have each group play their own grouped rounds in an assigned month that most closely aligns with the seasonality of the sport and temperature of play. We assume that the ideal temperature for play is a daily average of 20 degrees celsius. We measure how suitable a particular month by how small the difference Taverage-Tideal between the average temperature of a region in a particular month (the average of all countries in a region) and the ideal temperature for play, which we call the **cost**. To find an optimal assignment of regions to months, we evaluate how suitable an assignment is by adding up the differences between average and ideal temperatures for each region and we use the assignment which minimizes this **sum**.

In order to minimize this sum we use the well known "**hungarian algorithm"** (CP Algorithms, 2023) which can be used to solve the problem of **optimal assignment.** We form a cost matrix M where Mi,j represents the cost of team i playing in the jth month.

<table>
<tr><th></th><th>FEB</th><th>MAR</th><th>APR</th><th>MAY</th><th>JUN</th></tr>
<tr><td>ASIAN PACIFIC</td><td>| 12.50 - 20 |</td><td>| 14.00 - 20 |</td><td>| 15.50 - 20 |</td><td>| 16.75 - 20 |</td><td>| 17.75 - 20 |</td></tr>
<tr><td>CENTRAL AMERICAN</td><td>| 17.25 - 20 |</td><td>|17.75 - 20 |</td><td>| 18.75 - 20 |</td><td>| 18.25 - 20 |</td><td>| 17.75 - 20 |</td></tr>
<tr><td>SOUTH AMERICAN</td><td>| 24.75 - 20 |</td><td>| 23.25 - 20 |</td><td>| 21.50 - 20 |</td><td>| 17.75 - 20 |</td><td>| 16.00 - 20 |</td></tr>
<tr><td>NORTHERN EUROPEAN</td><td>| 05.00 - 20 |</td><td>| 07.75 - 20 |</td><td>| 11.00 - 20 |</td><td>| 14.25 - 20 |</td><td>| 18.75 - 20 |</td></tr>
<tr><td>MEDITERRANEAN</td><td>| 12.50 - 20 |</td><td>| 14.50 - 20 |</td><td>|17.00 - 20 |</td><td>| 20.25 - 20 |</td><td>| 23.00 - 20 |</td></tr>
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ASIAN PACIFIC	CENTRAL AMERICAN	SOUTH AMERICAN	NORTHERN EUROPEAN	MEDITERRANEAN
MARCH	FEBRUARY	APRIL	JUN	MAY

Part 3: Time of day

We will put each game on a weekend. We also look at:

- Local time in the home country
- Local time in the away country
- Fan engagement in the home and away countries.
    - Fan proportion index
    - Fan population index
- Home country priority (see explanation below).

Viewership in the home country will be prioritised.

Part 3: Combined fan index

We generate a combined fan index for each of the two countries playing with this formula:

$$Fan\ Index\ Home = \frac{0.78}{0.63 + 0.78} \cdot I_{fan\ proportion} + \frac{0.63}{0.63 + 0.78} \cdot I_{fan\ population}$$

We will calculate Fan Index Away with the same formula. Note that the values of Ifan proportion, Ifan population, and 0.78, 0.63 were taken and generated in **section 2.2.2**, and the indexes of fan proportion and fan population will align with data of that particular country.

For a particular time slot of one hour, we will weigh the TV view percentage of the two countries at that time slot with their respective fan index, in addition to the country weight.

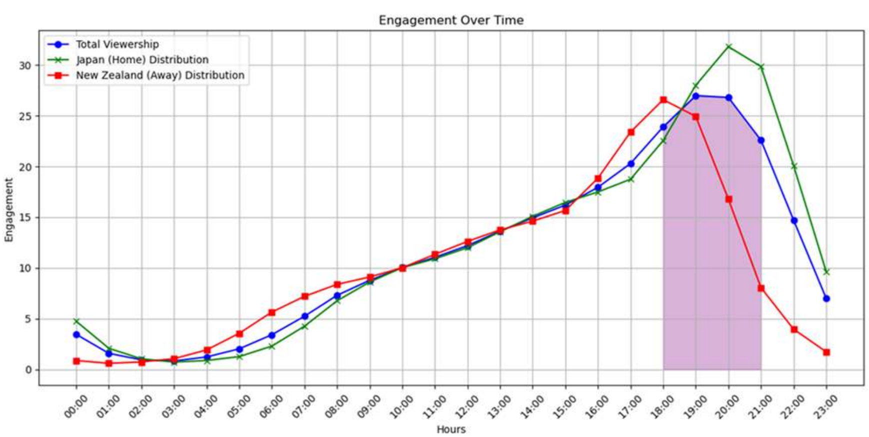
$$Total\ Viewership = \frac{2}{3} \cdot \frac{Fan\ Index\ Home}{Fan\ Index\ Home + Fan\ Index\ Away} \cdot V_H + \frac{1}{3} \cdot \frac{Fan\ Index\ Away}{Fan\ Index\ Away + Fan\ Index\ Home} \cdot V_A$$

Where we see that V_H is the percentage of the population that watches TV at this particular time slot in the home country, similarly for VA.

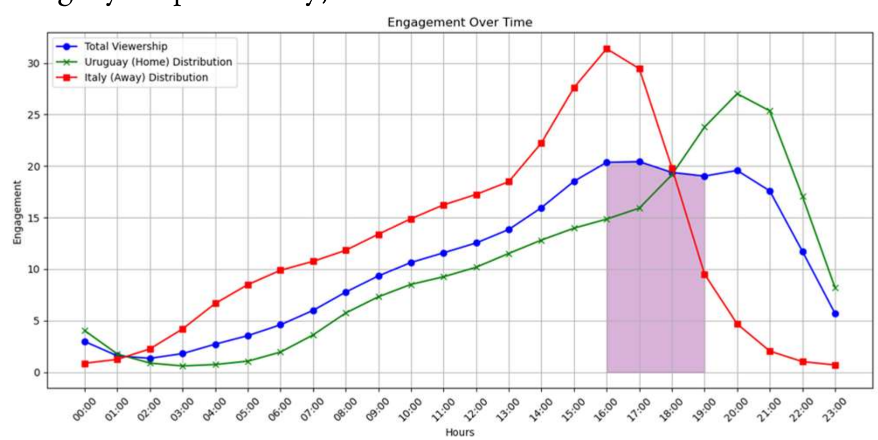
Repeat this process 24 times for each of the 24 one hour periods during the day.

Part 3: Timing example

a particular match between Japan (home) and New Zealand (away). Note that these two countries are in the same group, and by our clustering design, the time zone difference between the two is not significant. Most of the TV viewership appears near the "prime time" of around 7pm, so our model chose 6-9pm Japan time as the scheduled time to balance viewership in Japan and New Zealand.



a hypothetical matchup between Uruguay (Home) and Italy (Away), which could happen in our qualifier round robin. The time zone difference between these two is more extreme, as shown by the two different peaks (red & green) in the figure. This results in a plateau of the total viewership from 4pm to 8pm as all plausible starting times, however our model chose 4pm Uruguayan time due to the higher fan index in Italy. (4pm in Uruguay is 9pm in Italy).



The time on the x-axis is based on the time in the home country.

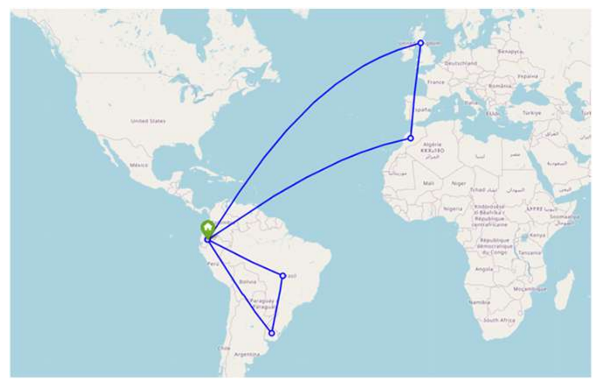
Part 3: Simulating Future Scheduling

After the grouped round robin is completed, a qualifier round robin (2nd and 3rd from the groups) will take place, which we explore in the next paragraph. A placement round robin is also held for the 4th country from their respective groups and will follow a similar format as the round robins before. In terms of the knockout round for the top 8, we will be scheduling them in a centralised location after the round robins are finished. This is because in doing so, we raise excitement and engagement for the best 8 teams in the league, as fans across the world travel to the centralised location to support. To determine this location, the participating countries will follow a bidding system much like the Olympics.

Due to the nature of our format, the qualifier and placement round robins are unable to be scheduled beforehand. Instead, we provide a theoretical pool of 10 teams for the knockout qualifier round, and show how our scheduling algorithm is able to produce accurate results for all possible future rounds, regardless of the result. Using our **simulated annealing** algorithm, we have created a schedule for the teams that qualified for the knockout. The matches of Ecuador and its team's flight path is shown below.

Ecuador [1911]

vs	Italy	home	[1914]
vs	Morocco	away	[1807]
vs	England	away	[2012]
vs	France	home	[2031]
vs	Mexico	home	[1817]
vs	Uruguay	away	[1922]
vs	Brazil	away	[1994]
vs	South Korea		[1745]



Expansion and addition of teams

In accordance with our goals for team selection and the index we developed to fulfill them, the next four teams will be the four following Italy on our **priority index**, which are Turkey, Switzerland, Norway and the Netherlands.

Determining new groups

To determine the clusters for our 24 countries, we will run the same simulation as previously. The number of clusters against the IF value is presented below.

No. of Clusters	2	3	4	5	6	7	8
I_F	0.391	0.453	0.564	0.652	**0.762**	0.640	0.398

Additional details after addition

Our tournament format will be adjusted for the addition of the new teams. Specifically, after the top team in each group stage qualify for the knockouts, the 2nd and 3rd team in each group (this will be 26 = 12) teams will be in the qualifier round robin, and only the top two in this round robin will advance to the knockouts. The data for this model after 1000 simulations is presented in the following table.

T_g	T_a	σ_N	P_A	L	I_F
124 Games	1901 Hours	0.80	57.8 Hours	5.36	0.712

To look at the number of games that each team plays, we get that each team will play an average of 10.3 games each. The minimum number of games that a team will need to play in order to be crowned champion stays the same at 6, while the upper bound has increased to 17, due to the need for the team to play 2 extra games in the knockout qualifier, as we have 12 teams in it instead of the 10 originally.

Sensitivity analysis

We conduct sensitivity analysis to two parts in our model, where we used AHP to create the weights for our indexes. Using the **One at A Time (OAT)** method, we perturb each weight by a **fixed percentage of 10%**. We will then analyse the effects that these changes had on our priority index and format adequacy index.

Sensitivity Analysis on the selection of the 20

We will look at the effects of increasing/decreasing the weights used to determine our **selection of 20 teams** for our league and how that changes the teams that we choose. **We increased/decreased each of the weights for the indexes** I_{skill}, $I_{fan\ proportion}$, $I_{fan\ count}$, $I_{funding}$ and I_{green} one by one. We found that no changes were made to our selection of 20 teams seven times out of ten. The three times a team was swapped out, we found that Egypt was replaced with another African country, that being Senegal or Keyna. However, we can see from the priority table that these countries have very similar index scores, making it unsurprising. There was only one instance where two teams were swapped out, that being Egypt with Senegal and Italy with Switzerland. Again, from the priority table we can see that Switzerland was highly ranked to begin with and Italy was the lowest ranked out of the 12 teams chosen after picking 2 teams from each continent. This means that the model we used to select our initial 20 teams is very accurate and our selection of weights is appropriate.

Sensitivity Analysis for the decision of the tournament formats

We will look at the effects of increasing/decreasing the **weights** on each of the variables T_g, T_a, σ_N, P_A, L, I_F by 10% on the format adequacy index IF. It is worth noting that regardless of the change on the weights, the preferred format stays the same as #5 Hybrid Grouped Single Round Robin + Knockouts - (by region + size). However, there were particular changes to the weights that affected the dominance of the #5 format, and these changes had been extracted from the appendix and presented below:

	#1	#2	#3	#4	#5	#6	#7
T_a: Decreased	0.721	0.483	0.636	0.531	**0.725**	0.632	0.616
L: Decreased	0.708	0.449	0.649	0.533	**0.732**	0.629	0.645

First, note that the closest contender to format #5 is format #1, full single round robin. When T_a's weighting was decreased, there was a much closer gap between the two, and when the weighting for L was decreased, the dominance of the #5 format was much more apparent. This makes intuitive sense, as when each teams play every other team as in #1, the loss L is minimised, but this comes at a cost of greater T_a. So when the weight for L is decreased, the advantage of #1 in the L category is diminished, thus #5 becomes the clear winning format. Similar argument can be made for the change in T_a.

Generalisation of the model

When generating a schedule for other team sports, special considerations might be taken for how a sport works in a tournament setting.

- How are points typically allocated in different sports?
    - In volleyball, points are often assigned based on the number of 'sets' a team wins. Our model still runs effectively, as the round robin system can be ranked based on the rules of the sport.
- How long do matches last?
    - For a majority of team sports, any typical game length (1-5 hours) will not affect our model, as we could change the **time scheduling model** to take the greatest total viewership over that corresponding time length.
    - However, some sports like Test Cricket can be played for days, which a generalised model should consider in terms of timetabling and required rest.
        - Our model can be adapted for situations like these, where weights can be adjusted to prioritise lower numbers of games, which would naturally allow for more time between matches and allow athletes to rest.
- Different ranking systems in other sports
    - When generating a schedule for other team sports, elo ratings can be used similarly to football to rank teams and predict match outcomes. These team sports have their own ranking systems which we assume can accurately represent a national team's competitive strength just like the World Football Elo Ratings. For sports like basketball, Elo ratings can be adapted (eg. FIBA rankings for international teams), but sports without elo systems, like volleyball, may require alternative metrics such as world rankings or recent tournament performance.
- Viewership Patterns
    - Generalising our model to accommodate for other team sports would need data on specific viewership statistics for the particular team sport. We would implement the data into our model and predict peak viewership times and optimal patterns.
    - For different sports with different ideal playing times, our model is able to adapt when deciding the time in the day to host games.
- Scheduling
    - The model should adjust for sport-specific seasons, weather constraints, and cultural events, while optimising time slots for peak viewership based on the sport's audience patterns.

Conclusion and Discussion

Assessment of our model

Our model is capable of providing an accurate schedule for any given sport and event. However, it is not perfect. Where our model is flawed is its lack of generalisation for all sports as a scheduling model, as the weights in the AHP need to be manually adjusted for the specific goals of the tournament. However, given to any scheduling body with sufficient data, the model is able to effectively select teams and generate a schedule.

Strengths

- Our format selection algorithm is comprehensive and accurate, with generated weights able to reconcile a broad range of comparative importances. Here, it is easily adjustable to the specific needs of any tournament organiser.
- Our models for format selection and matchmaking are highly considerate of athletes and different goals of the tournament organisers and is able to maximise the fans' experience when watching our tournament from all around the world.
- Our models are highly resistant in our sensitivity tests, making them far more reliable.

Weaknesses

- The team selection model requires specific data for each national sports team and nation that is trying to enter the tournament. In practicality, if a central database was not found for the sport, our proposed team selection process could become time consuming.
- Our matchmaking algorithm to allocate team matchups on each round is not designed to create the perfect or ideal matchup, but runs until an approximate solution is found that is reasonably close to the ideal. This algorithm can be time consuming and require constant monitoring to be effective.

Conclusion

The task provided to us in this paper was to create a quantitative and generalised model for the scheduling and matchmaking of a global sports league. What we have done is create a set of adaptable algorithms that can be used to model for any sport league, selecting optimal teams, choosing ideal tournament formats, and timetabling them to appropriate times. The objectives outlined in the task were to ensure equitable numbers of games, travel distances, and matchups. We have completed that, as well as ensuring that a plethora of other necessary considerations were made to create a highly suitable scheduling model.

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