



IMMC 2025 BATORY HS

OBJECTIVES



Generalization

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Sport



Teams



Scheduling



Generalization



Sport

Key aspects:

- ❖ **Global fanbase**
- ❖ **Playing frequency**
- ❖ **Competition level across continents**
- ❖ **No dominant nations**

**Fan Volume
Index
(FVI)**

**Match
Fairness
Index
(MFI)**

Fan Volume Index (FVI)

$$\text{Fan Volume Index (FVI)} = n \times f$$

Where:

- ❖ n – number of fans of the sport worldwide
- ❖ f – number of times per week the sport can be played

Match Fairness Index (MFI)

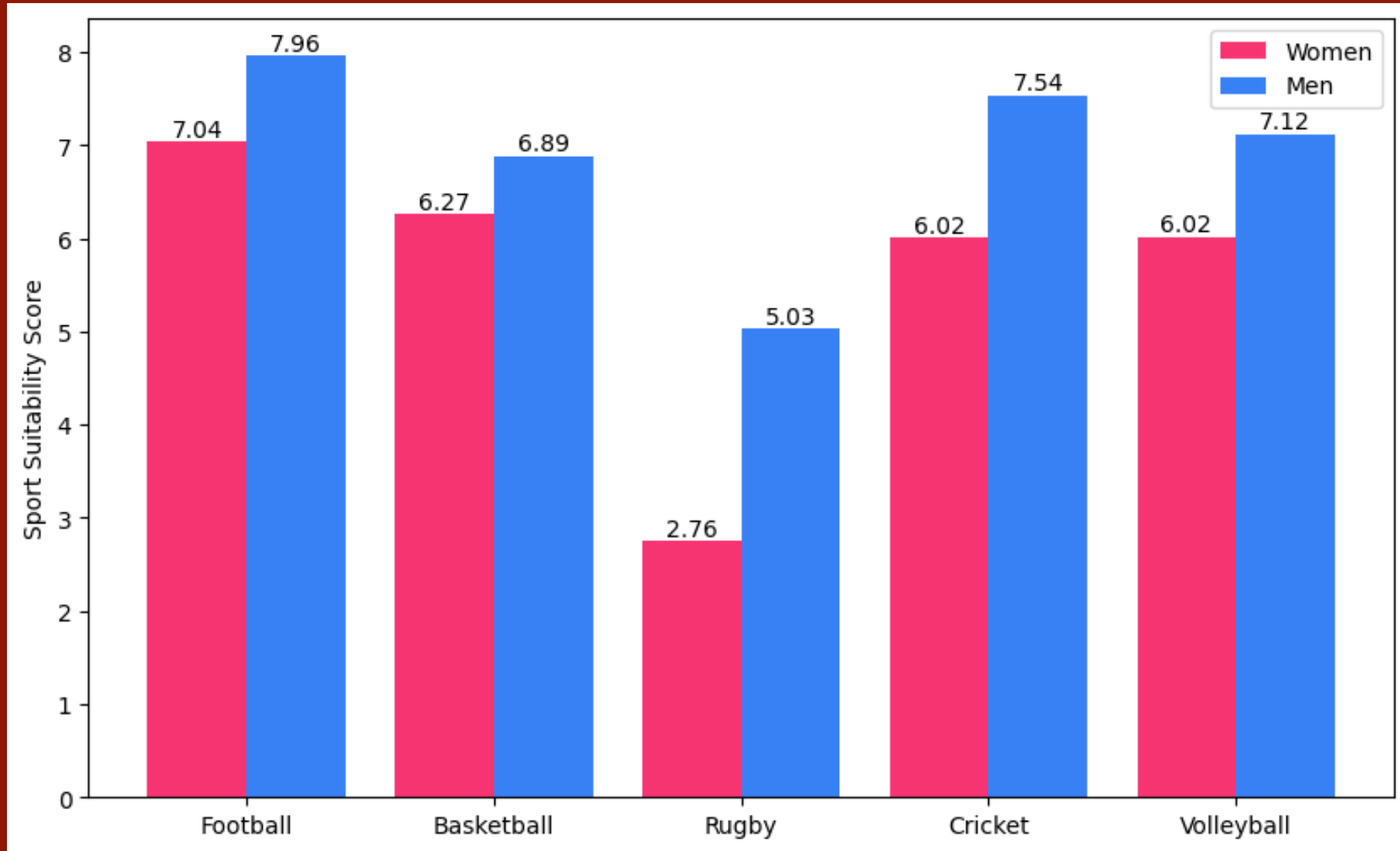
$$\text{Match Fairness Index (MFI)} = \frac{v}{\sigma_{\text{continents}}}$$

Where:

- ❖ v – number of different winners in quadrennial world championships
- ❖ $\sigma_{\text{continents}}$ – standard deviation of the number of teams in the top 50 global ranking per continents

$$\text{Sport Suitability Score (SSS)} = \ln(\text{FVI} \times \text{MFI})$$

SSS Results





Sport

- ❖ **Global fanbase**
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ELO rating system

**National league
matches**

**International
competition
matches**

Taking into account different league strengths

Team win probability

$$E_{T1} = \frac{1}{1 + 10^{\frac{R_{T1} - R_{T2}}{400}}}$$

Where:

- ❖ E_{T1} - probability of Team 1 winning
- ❖ R_{T1} - ELO ranking of Team 1
- ❖ R_{T2} - ELO ranking of Team 2

$$\text{league factor (lf)} = \frac{s_{\text{league}}}{s_{\text{benchmark}}}$$

Where:

- ❖ s_{league} - league relative strength score
- ❖ $s_{\text{benchmark}}$ - lowest strength score in the ranking

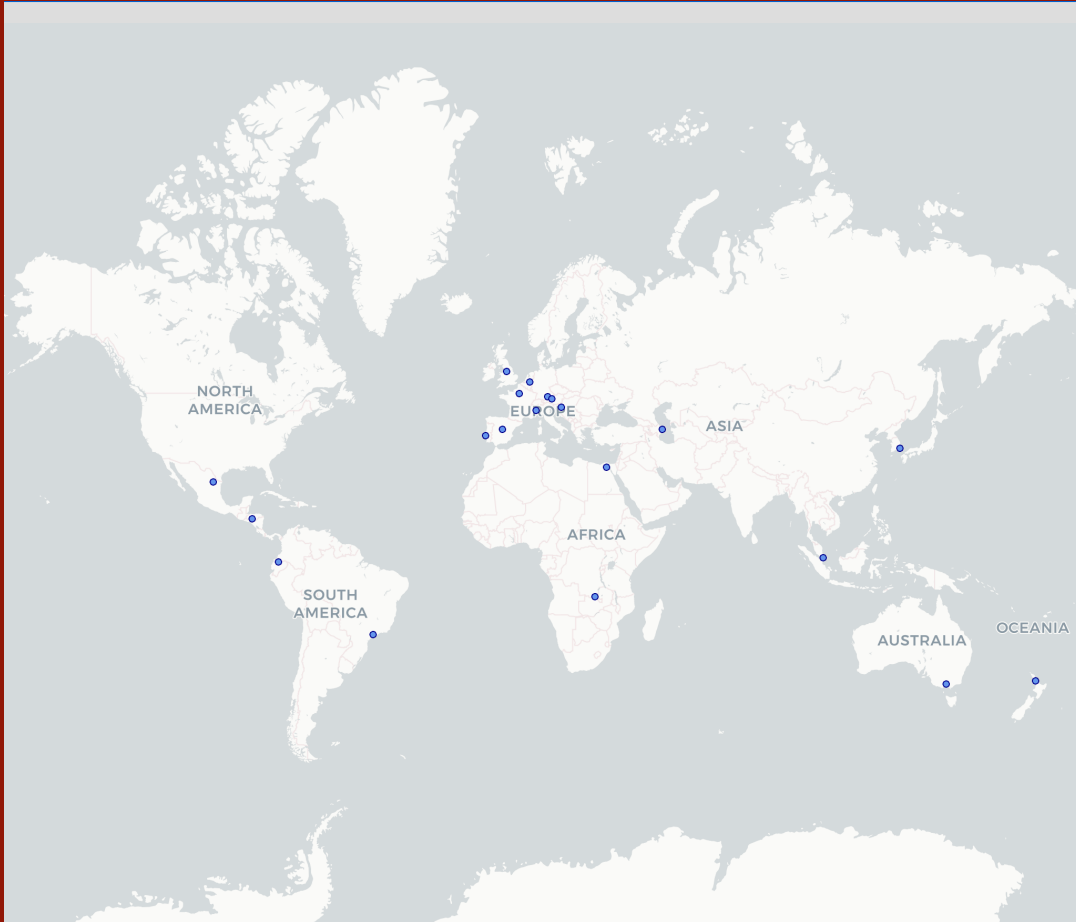
International competitions bonus

$$\text{international factor (if)} = \ln\left(\frac{p_{\text{competition}}}{1000}\right)$$

Where:

- ❖ $p_{\text{competition}}$ - prize value of the competition

Chosen teams



- ❖ **Real Madrid**
- ❖ **Bayern Munchen**
- ❖ **Manchester City**
- ❖ **Dinamo Zagreb**
- ❖ **Paris Saint-Germain**
- ❖ **Benfica**
- ❖ **SE Palmeiras**
- ❖ **RB Salzburg**
- ❖ **Juventus**
- ❖ **Al Ahly**
- ❖ **PSV Eindhoven**
- ❖ **Qarabak FK**
- ❖ **TP Mazembe**
- ❖ **LDU Quito**
- ❖ **Johor FC**
- ❖ **Ulsan Hyundai FC**
- ❖ **Auckland City FC**
- ❖ **UANL Tigres**
- ❖ **CD Olimpia**
- ❖ **Melbourne Heart FC**



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Schedule's goal

- ❖ **Minimize Travel distance**
- ❖ **Minimize Travel distance inequalities**

$$\text{Minimize Cost} = W_{Total} \cdot TotalTravel + W_{StdDev} \cdot StdDev(Travel)$$

- ❖ **Constraints:**
 - ❖ **Play Everyone Twice (Double Round Robin)**
 - ❖ **One Game Per Week**
 - ❖ **Teams can't stay home or away forever**

Financial analysis

Costs considered:

- ❖ **Hotel stays**
- ❖ **Emissions**
- ❖ **Flights**
- ❖ **Match organization**

$$Team + Environment (total) = 2,785.3364 \times d + 1,473,070 \times n + 1,180 \times w [USD]$$

Obtaining a DRR schedule

Step 1:

CP-SAT solver - a constraint programming tool (CP) that uses different satisfiability (SAT) techniques to obtain fitting results:

- ❖ **The two teams teams play in either of their respective cities**
- ❖ **Every team hosts a match with all the teams once a season**
- ❖ **Enforce the max consecutive home / away games - set to 3**

Step 2:

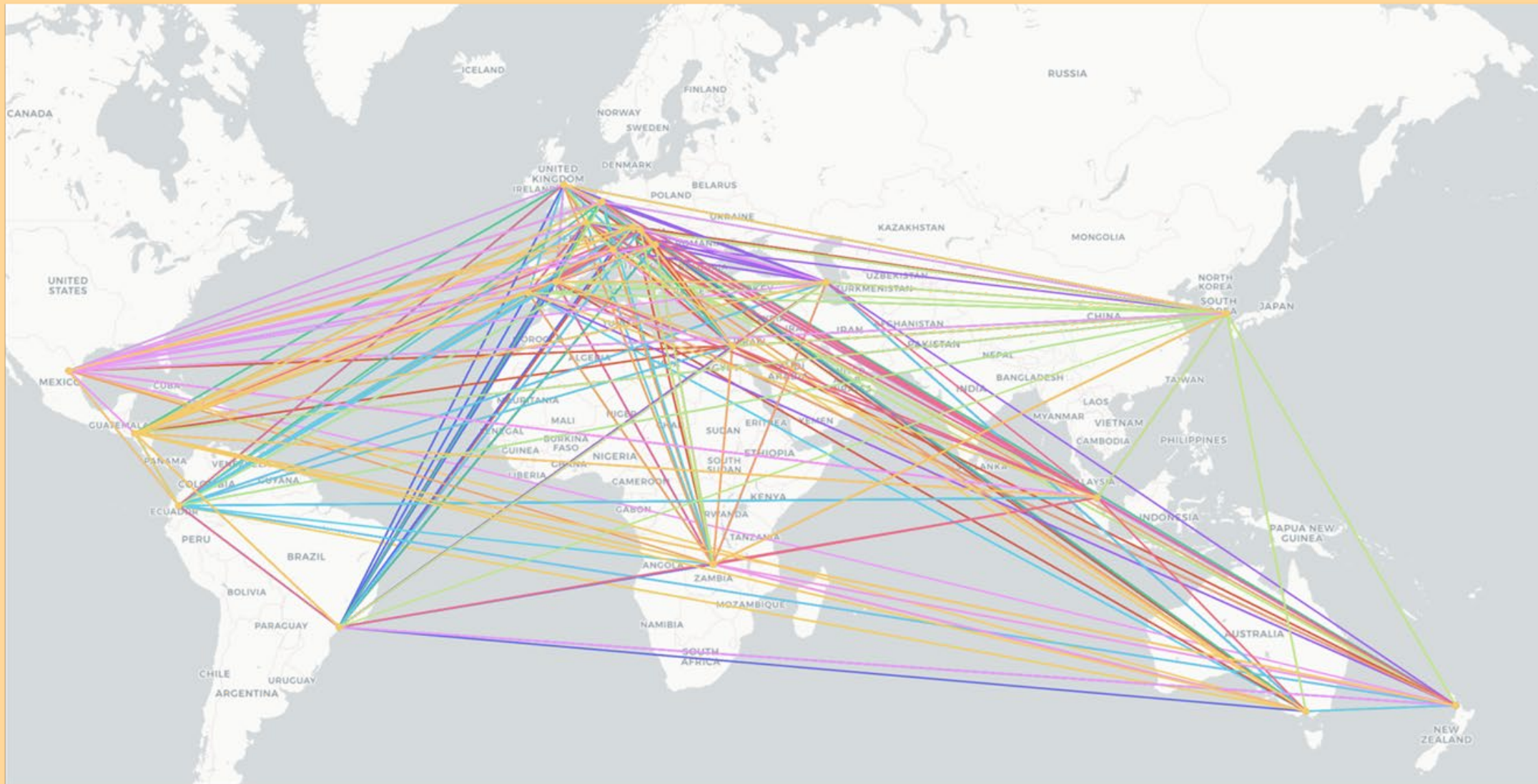
Simulated Annealing (SA) - a probabilistic technique used for finding the global optimum of a function

Optimization results

Metric	Initial Feasible Schedule	Optimized Schedule
Total Travel Distance (km)	4,935,726	4,501,332
Std. Dev. of Team Travel (km)	61,532	13,455
Spread of Team Travel (km)	244,367	42,645
Min Team Travel (km)	165,372	209,740
Max Team Travel (km)	409,739	252,385
Combined Cost (Eq. 10)	70,613,073	17,803,059

Table 1: Comparison of Initial (CP-SAT) and Optimized (SA) Schedule Travel Metrics

Travel path visualization



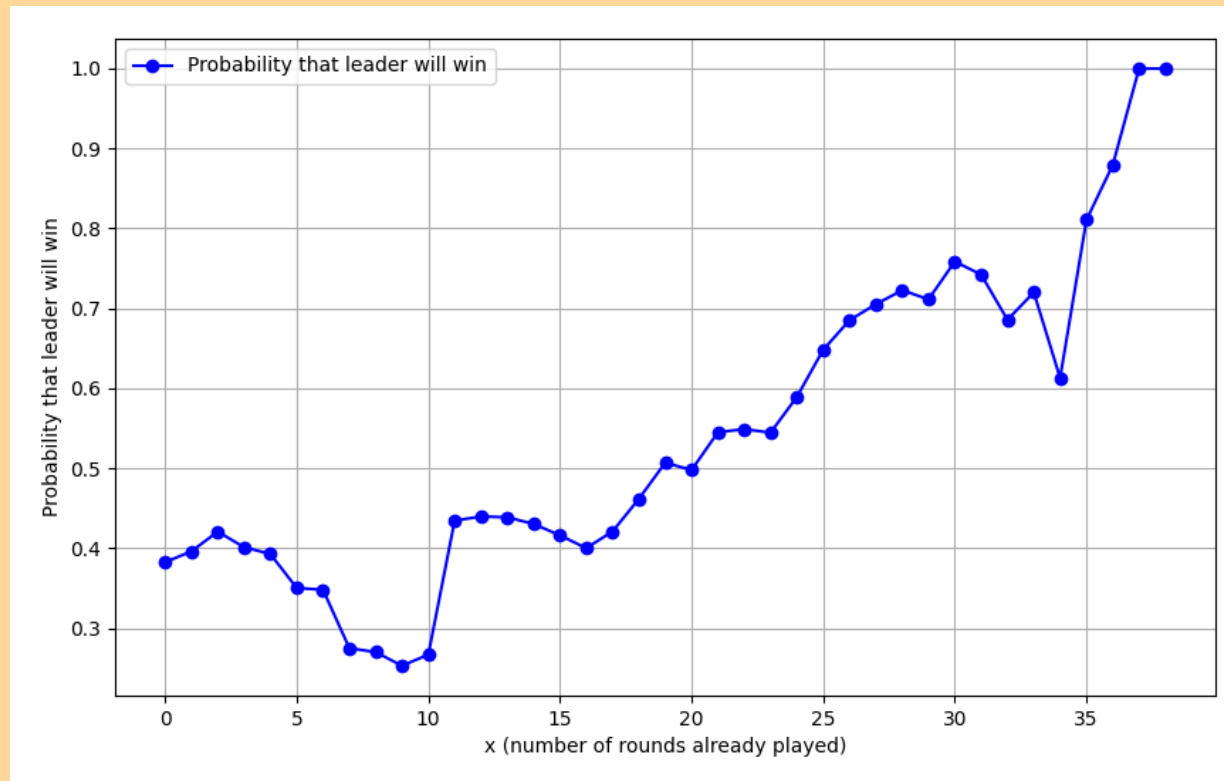
Teams

- Real Madrid
- Bayern München
- Manchester City
- Dinamo Zagreb
- Paris Saint-Germain
- Benfica
- RB Salzburg
- Juventus
- Auckland City FC
- Melbourne Heart FC
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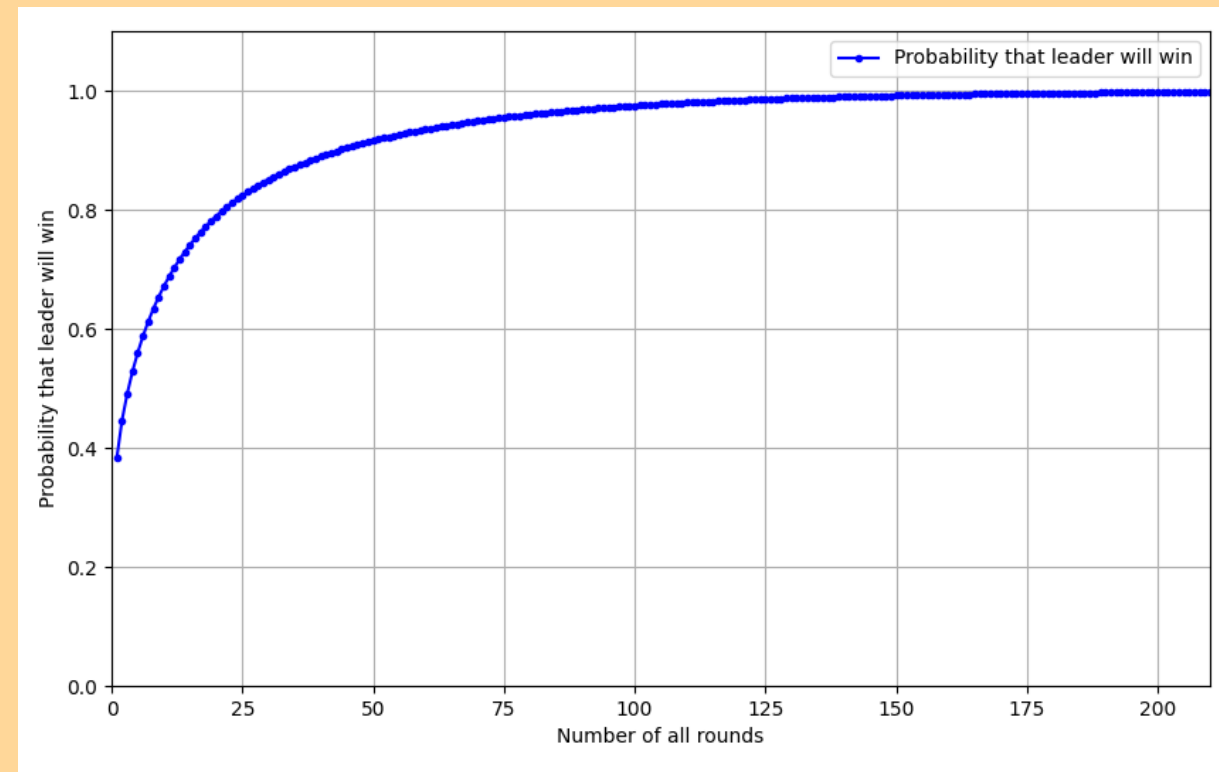
Is the win certain?

- ❖ Assuming that the team's final score undergoes normal distribution we can calculate expected value of it's final score
- ❖ Using a cumulative distribution function we can predict probability that the team with the highest elo rating will win

Probability that L will win after each round (10 000 simulations)



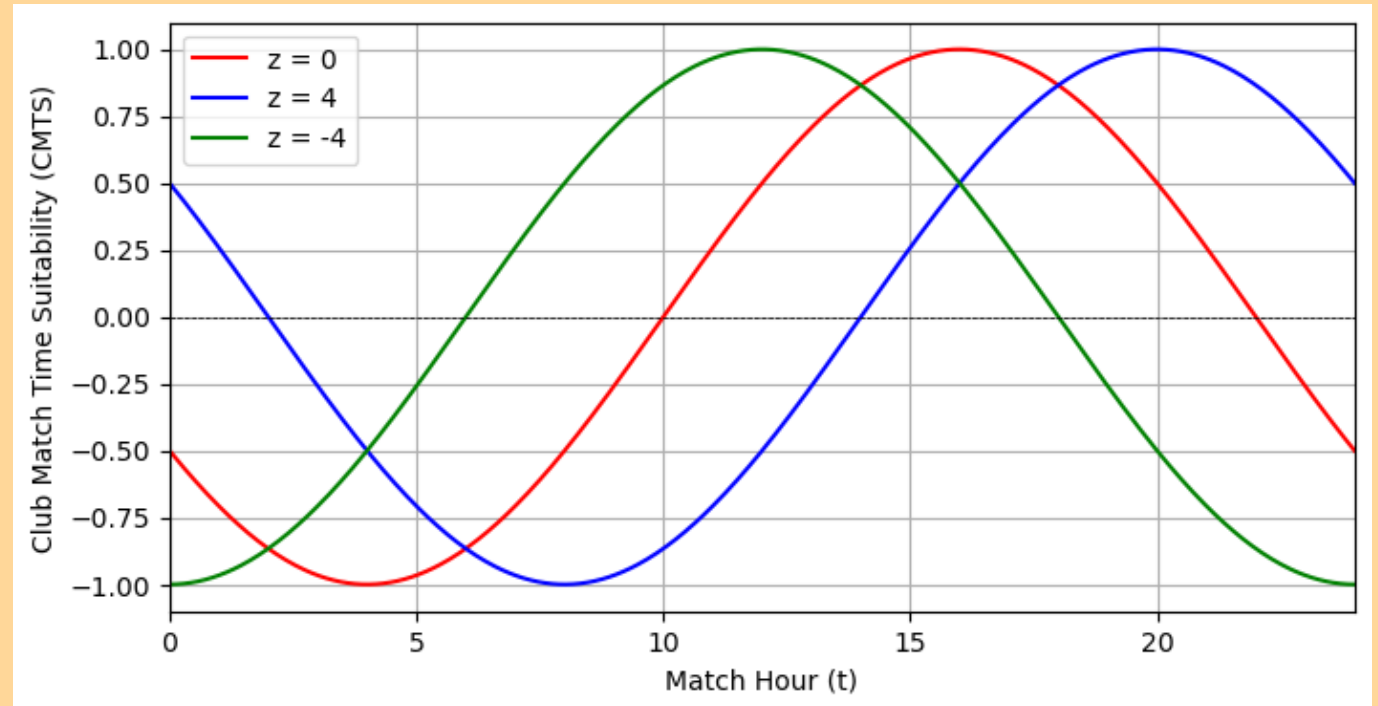
Probability that L will win in case of x number of rounds



Generating match hours

- ❖ **Human body can shift its biological clock by 1 hour a day**
- ❖ **Team gets to the city they play in 4 days before their match**
- ❖ **2 teams playing experience the same level of discomfort**

$$\begin{aligned} &\text{Club Match Time Suitability (CMTS)} \\ &= -\cos\left(\frac{2\pi}{24} \times (t - 4 - z)\right), 0 \leq t < 24 \end{aligned}$$



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Extending the GSL

Added teams:

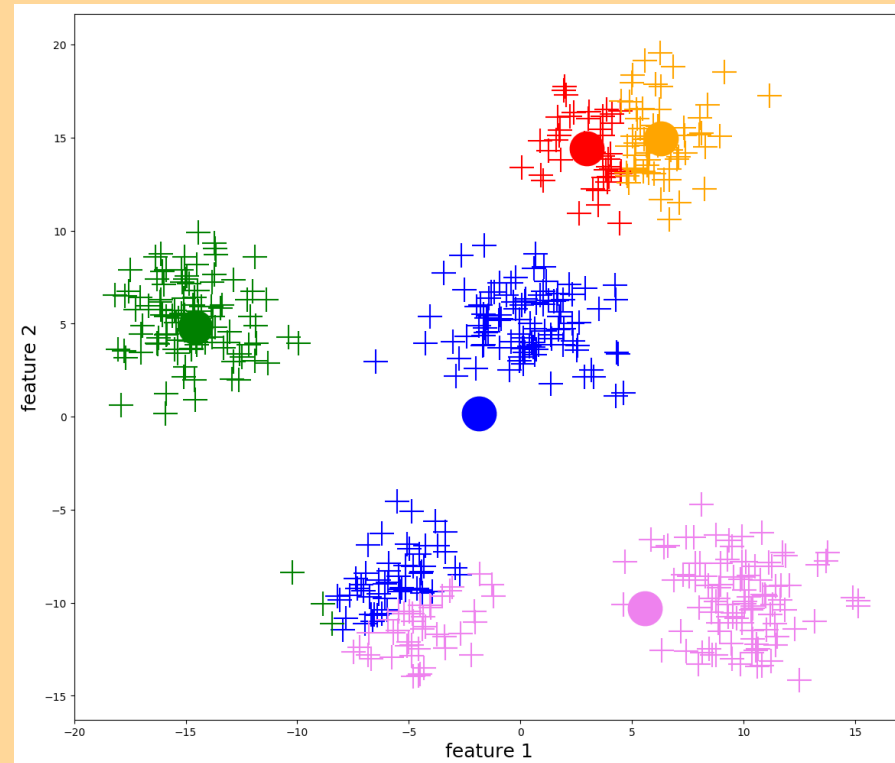
- ❖ **Shaktar Donetsk**
- ❖ **Mamelodi Sundowns**
- ❖ **Celtic FC**
- ❖ **Esperance Tunis**



Double Round Robin takes too much time



K-means algorithm



**1. Random
centroids**

**2. Assign
points
closest to
centroids**

**3. Arithmetic
mean of
centroids
create new
centroid**

**4. Iterate
steps 2 and
3**

Our clustering algorithm

Key assumption: each cluster has 4 teams

$$\vec{v_i} = (\underbrace{x_i}_{\text{Vector}}, \underbrace{y_i, z_i}_{\text{Geography}}, \underbrace{w \times s_i}_{\text{Weight} \times \text{score}})$$

Distance between cities i and j in 4D

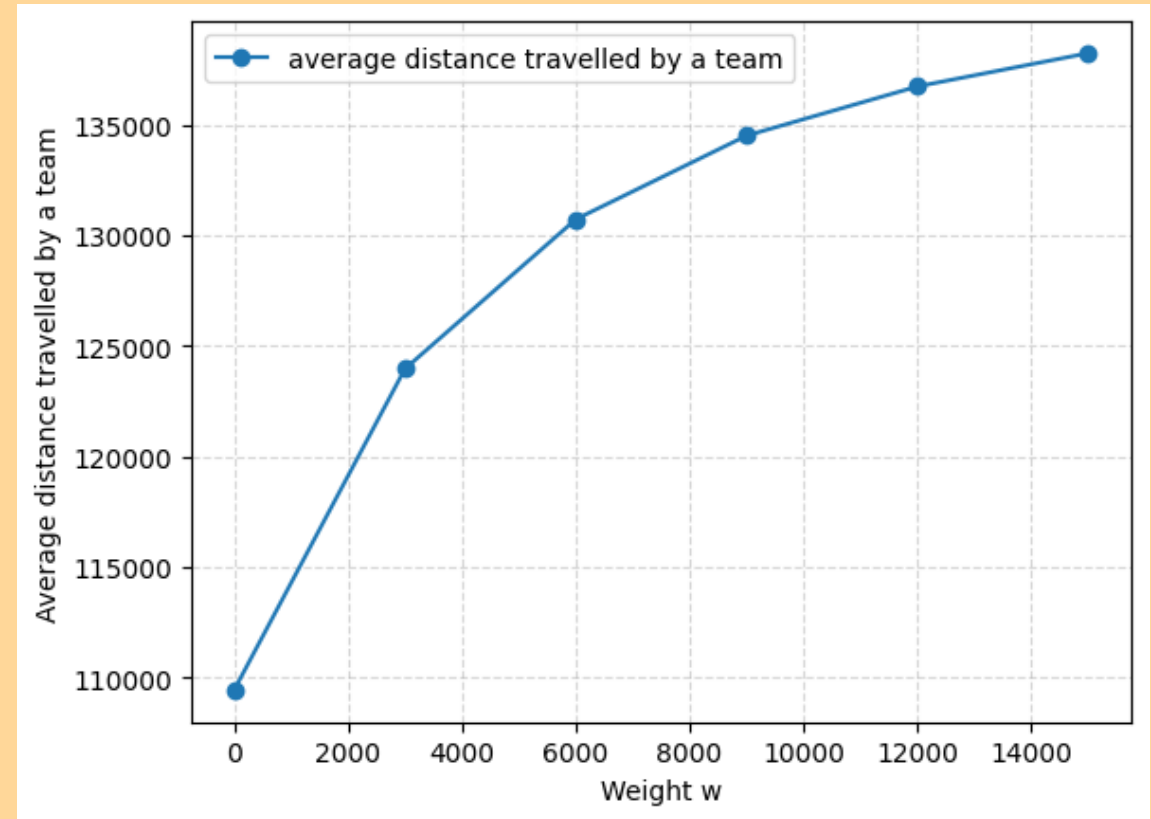
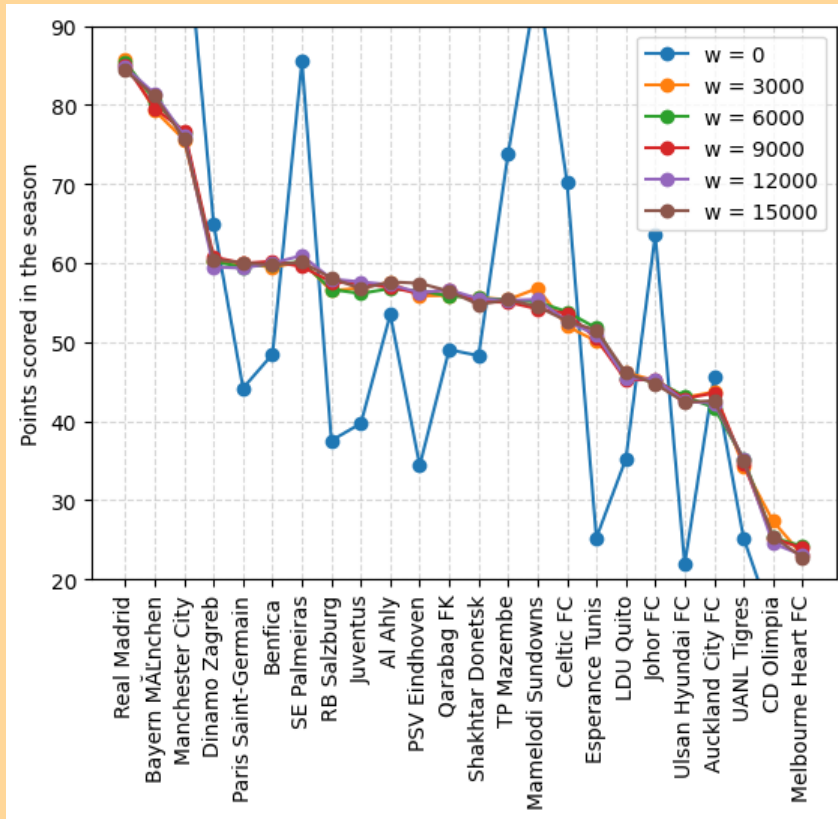
$$D(v_i, v_j) = \sqrt{\left(d_{geo}(x_i, y_i, z_i, x_j, y_j, z_j)\right)^2 + w^2(s_i - s_j)^2}$$

Where:

$$d_{geo}(x_i, y_i, z_i, x_j, y_j, z_j) = 6371 \arccos \left(\frac{x_i x_j + y_i y_j + z_i z_j}{6371} \right)$$

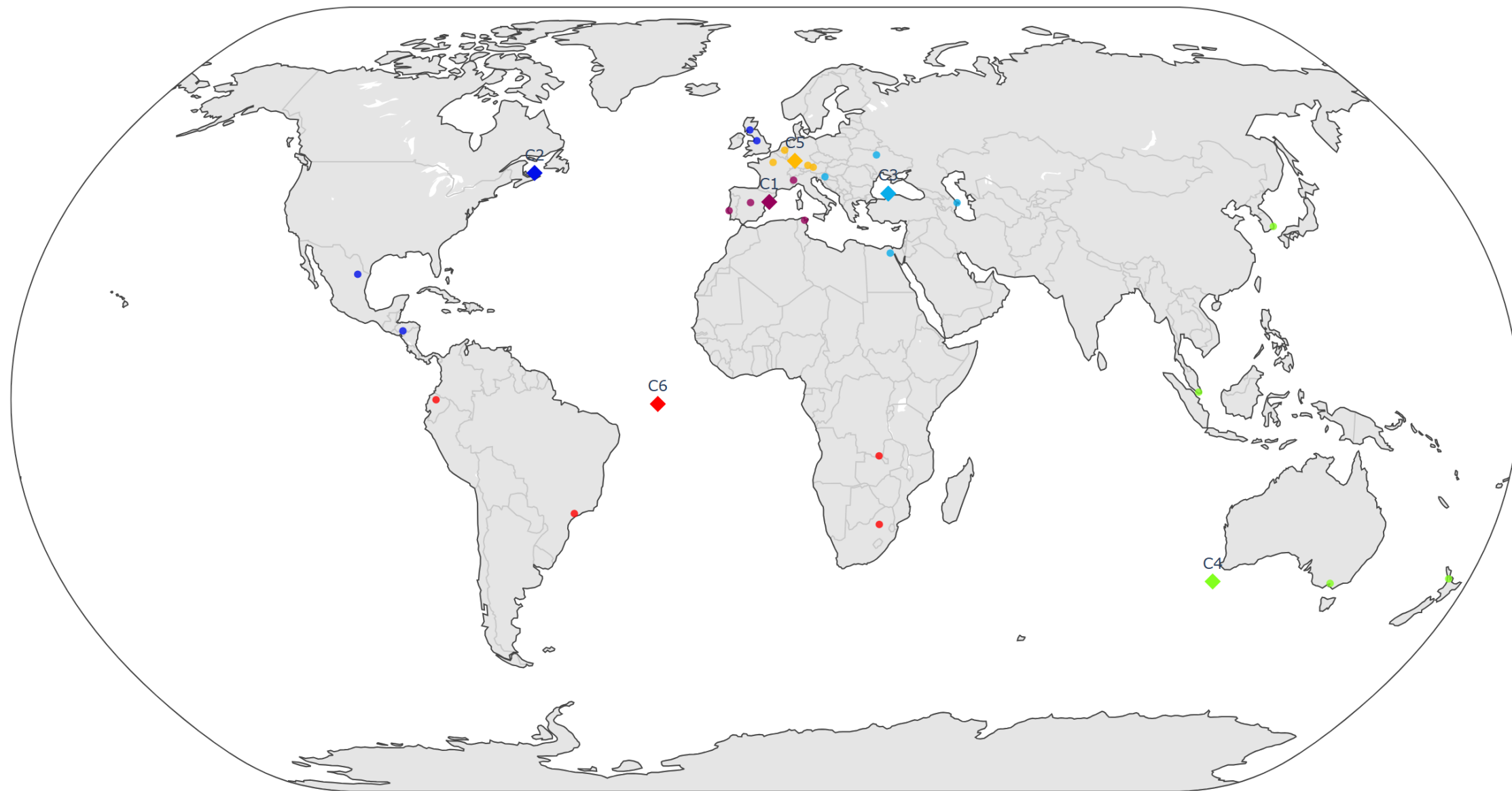
Choosing the weight w

Trade-off between goals



$w = 9000$

Sample clusters



K - means results

Metric	Optimized Double Robin Schedule	Balanced K-means schedule
Total Travel Distance (km)	4,501,332	3,376,193
Distance per team (km)	225,067	140,674
Mean ELO difference	380	192
Number of match days	38	36
Number of teams (km)	20	24
Estimated total cost (\$)	65 mln	70 mln.

Increase in cost is caused by an increase in number of teams

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Extending the model to other sports

- ❖ **Our model can be extended to other zero-sum games - one's person loss is equivalent to another person's gain.**
- ❖ **Key requirements;**
 - ❖ **Sufficient datasets (historical games, league rankings, etc.),**
 - ❖ **Adjusting the ELO system according to the rules of the sport,**
 - ❖ **Ensuring that the sport fits all the key assumptions of the model**



**THANK YOU
FOR YOUR
ATTENTION**